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Destruction of the Kondo effect by a local measurement

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Abstract

We show that a local measurement which decoheres the Kondo center in a Kondo system, suppresses the Abrikosov–Suhl resonance and destroys the Kondo effect. This happens due to elimination of the entanglement between the Kondo center and the conduction electrons, and differs essentially from smearing of the resonance by dissipation. Considering decoherence by a spin bath, we predict that the Kondo effect disappears when the Kondo temperature becomes smaller than the coupling with a bath. Suppression of the Kondo effect can be detected in experiments on "quantum corrals" or quantum dots doped by impurities with internal degrees of freedom.

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The Kondo effect discovered originally as a mechanism to explain the resistivity minimum in dilute magnetic alloys [1] is one of most interesting many-body phenomena in condensed matter physics. It plays a crucial role for heavy fermion physics [2], for metallic glasses and systems with orbitally degenerate ions [3], for quantum dots [4,5], for quantum tunneling in metals [6], etc. The key feature of the Kondo effect is the formation of the Abrikosov–Suhl (AS) resonance near the Fermi level. The STM technique allowed visualization of this resonance for magnetic impurities [7,8] and for the orbital Kondo effect on atomically clean metal surfaces [9]. The AS resonance is a consequence of the strong quantum correlations between the Kondo center (localized magnetic moment, or other entity with internal degrees of freedom) and the conduction electrons. These two subsystems form an entangled (singlet, for spin systems) state analogous to the state of the Einstein–Podolsky–Rosen spin pair [10]. Such states are often fragile. If a quantum measurement is performed, which uniquely determines the state of one of the subsystems (or both), then the quantum correlations are destroyed. The measurement can be performed either by a specially designed device or

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by an environment, which can act as a measurement apparatus [10,11].

In this Letter we show that a measurement which decoheres the Kondo system, and destroys correlation between the Kondo center and the conduction electrons, suppresses the AS resonance and thus leads to destruction of the Kondo effect. We demonstrate that this phenomenon differs from the suppression of the Kondo effect by dissipation (resulting, e.g., from external radiation) [12]; namely, in the former case, the spectral weight of the AS resonance is reduced, while in the latter case, the resonance broadens.

A physical origin of the quantum degrees of freedom leading to the Kondo effect can be different. Kondo centers with fluctuating charge state such as mixed valence ions provides a nice example. For Ce ions fluctuating between Ce^{4+} (no f-electrons) and Ce^{3+} (one f-electron), keeping in mind a large degeneracy of the ground state term for f^1 configuration ($N = 2j + 1 = 6$) an exactly solvable model of the Kondo effect basing on an infinite-N limit turns out to be very efficient [2,13]. We proceed with the infinite-U degenerate Anderson model with the Hamiltonian

$$H = P \sum_{k,\nu} \left[\epsilon_k c_{k\nu}^\dagger c_{k\nu} + \epsilon_f f_\nu^\dagger f_\nu \right] P + H_1, \tag{1}$$

$$H_1 = P \sum_{k,\nu} \left[V \left(c_{k\nu}^\dagger f_\nu + f_\nu^\dagger c_{k\nu} \right) \right] P, \tag{2}$$

where $c_{k\nu}$, f_ν are the Fermi operators for conduction and localized f-electrons, correspondingly, ϵ_k and ϵ_f are their bare energies counted from the Fermi level, V is the hybridization parameter, $\nu = 1, 2, \ldots, N_f$ is the “flavor” index and P is the projection operator into the space with $n_f = \sum_\nu f_\nu^\dagger f_\nu < 2$.

The essence of the Kondo effect, as described by this model, is the fluctuation of the charge of the impurity and the flavor of the electron localized on the impurity. Fluctuations of both these degrees of freedom are important. For $N_f = 1$, the flavor fluctuations are impossible, and the Kondo effect is absent (so-called Fano–Anderson model). Similarly, for fixed impurity charge state ($n_f = \text{const} = 1$), the amplitude of the AS resonance is zero, the Kondo temperature is zero, etc. If both degrees of freedom are measured (and, as a result of the measurement, become fixed), the maximum possible information about the impurity is obtained, and the impurity becomes completely “detached” from the Fermi sea. It is, at least intuitively, clear that the Kondo effect is destroyed completely after such a measurement. However, in real experiments, it is difficult to measure the flavor state. We show that by measuring only the impurity charge (determining only the number of f-electrons, $n_f = 0$ or $n_f = 1$), almost the same result can be achieved. The Kondo effect becomes strongly suppressed, and the amplitude of the AS resonance diminishes exponentially in $V^2 N_f$. Therefore, experimental check of the effect predicted here becomes feasible.

The infinite-U Anderson model is exactly solvable in the limit $N_f \to \infty$, assuming that $V \to 0$ and $V^2 N_f = \text{const}$. This approximation is realistic, and has been successfully applied to description of the electron spectra of cerium compounds [13]. The ground state wave function $|\Psi_0\rangle$ can be presented [13] as a superposition $|\Psi_0\rangle = A(|0\rangle + |1\rangle)$, where

$$|0\rangle = \prod_{\nu; k<k_F} c_{k\nu}^\dagger |\text{vac}\rangle,$$

$$|1\rangle = \sum_{k<k_F} |k\rangle \equiv \frac{1}{\sqrt{N_f}} \sum_{\nu, k<k_F} a_k f_\nu^\dagger c_{k\nu} |0\rangle, \tag{3}$$

where $|\text{vac}\rangle$ is the vacuum state, $A = \sqrt{1 - \langle n_f \rangle}$ is the normalization factor, $\langle n_f \rangle$ is the average occupation of the f-level, $a_k = V\sqrt{N_f}/(E_0 + \epsilon_k - \epsilon_f)$, and E_0 is the ground state energy counted from the energy of the “Fermi sea” state $|0\rangle$:

$$E_0 = \Gamma(E_0) \equiv V^2 N_f \sum_{k<k_F} \frac{1}{E_0 + \epsilon_k - \epsilon_f}. \tag{4}$$

The Green’s function of the f electrons $G(E) = \langle\langle f_\nu | f_\nu^+ \rangle\rangle_E$ has a pole at the energy $E^* = \epsilon_f - E_0$ with the residue $Z = 1 - \langle n_f \rangle$ which corresponds to the Abrikosov–Suhl resonance; the energy $E^* = \rho V^2 N_f (1 - \langle n_f \rangle)/\langle n_f \rangle$ (ρ is the density of states for conduction electrons) plays the role of the Kondo temperature provided that $Z \ll 1$; in that regime Z is exponentially small in $V^2 N_f$ (for more details, see, e.g., [2,13]). It is well known [14] that the AS resonance results in the Fano (anti)resonance in the conduction electron spectrum because of the identity

$$\langle\langle c_{k\nu} | c_{k'\nu}^\dagger \rangle\rangle_E = \frac{\delta_{k,k'}}{E - \epsilon_k} + \frac{V^2 N_f G(E)}{(E - \epsilon_k)(E - \epsilon_{k'})}. \tag{5}$$

It is the Fano antiresonance that is observed in the STM experiments [7,8].

The state (3) describes quantum correlations between the f-electrons and the conduction electrons. These two subsystems can be considered as an "EPR pair" [10], where the state of the conduction electrons is determined by the state of the f-electrons (and vice versa). One of the most impressive features of such states is that the decohering action applied to one subsystem immediately affects the state of the other subsystem, which has not been directly subjected to decoherence. We demonstrate below that in our case, such an influence, e.g., the measurement of the number of the f-electrons n_f, leads to an immediate change of the state of the conduction electrons, and suppresses the Kondo effect. This effect is similar to the situations explored recently for Anderson localization [15], in the Bose–Einstein condensate [16], and for an antiferromagnet [17]. However, the case of Kondo systems may be more easily studied in real experiments (see the discussion below).

Let us assume that the decohering influence of the apparatus is so effective that it can be described as a von Neumann's measurement [18]. This means that the initial density matrix $\rho_i = |\Psi_0\rangle\langle\Psi_0|$ is instantly transformed into the final one $\rho_f = A^2(|0\rangle\langle 0| + |1\rangle\langle 1|)$. The Green's function of the f-electrons after the measurement is

$$G_f(E) = -i\int_0^\infty dt\, e^{itE}\,\mathrm{Tr}\,\rho_f\big[f_\nu(t)f_\nu^\dagger + f_\nu^\dagger f_\nu(t)\big], \tag{6}$$

and can be evaluated as in Ref. [13]. One has to introduce the functions $\exp(-iHt)|\phi\rangle$ where $|\phi\rangle = |0\rangle, |k\rangle, f_\nu^+|0\rangle$, write the equations of motion for them, taking into account only the leading terms in $1/N_f$ and make the Laplace transformation. As a result, the Green's function after the measurement becomes (in the limit of large N_f):

$$\begin{aligned} G_f(E) &= \frac{A^2}{E - \epsilon_f + \Gamma(\epsilon_f - E)} \\ &= G(E) \\ &\quad\times\left[1 + \sum_{k<k_F}\frac{V^2 N_f}{\big(E_0 + \epsilon_k - \epsilon_f\big)(\epsilon_k - E)}\right]^{-1}. \end{aligned} \tag{7}$$

It has the same pole E^* as the Green's function before the measurement, but the residue is $(1 - \langle n_f\rangle)^2$ instead of $1 - \langle n_f\rangle$. Thus, the amplitude of the Abrikosov–Suhl resonance, and, consequently, the Fano antiresonance in the conduction electron spectra, diminish drastically after the measurement. Only a small fraction of the spectral weight, of order of $E^*/\rho V^2 N_f$ (which is of order of 10^{-4} for typical Kondo systems), survives. That is, the measurement strongly suppresses the Abrikosov–Suhl resonance, and practically destroys the Kondo effect. Experimentally, this effect can be checked with the "quantum corral" setup [8], by putting, e.g., a cerium atom on the metallic surface in the focus of an elliptic "quantum coral". Due to interaction with the f-electrons of Ce, the spectrum of conduction electrons exhibits the Fano (anti)resonance which can be observed by an STM tip placed at the other focus of the elliptic corral. The charge state of the Ce atom can be measured, e.g. by a point contact, placed near the atom (as has been analyzed in Ref. [19]), or by rf single-electron transistor. Such a measurement can be performed very fast, and, as the experiment-based estimates show, the picosecond-time measurement can be achieved [20]. In order to simulate the von Neumann's measurement (which is an ideal instantaneous process), the measurement time should be much smaller than $\hbar/T_K$ (typically, tens of picoseconds). Immediately after the measurement of the Ce atom, the amplitude of the Fano resonance should drop drastically.

It might be more feasible to employ the Kondo effect in quantum dots [4,5], where the bath of environmental degrees of freedom decoheres the central spin. It is also important, since the AS resonance is recovered during the characteristic time of order of $\hbar/T_K$ after the single measurement; in contrast, for a Kondo system continuously decohered by the bath, the recovery process is prevented. The bath works in essentially the same manner as a measuring device. Indeed, as the results above show, the spectral weight of the Abrikosov–Suhl resonance is determined by the non-zero value of the non-diagonal element of the density matrix $\langle 0|\rho|1\rangle = \langle 0|\Psi_0\rangle\langle\Psi_0|1\rangle \neq 0$ (see Eq. (3)). The interaction $\mathcal{V}$ between the dot and the bath reduces the value of $\langle 0|\rho|1\rangle$ [10,11], since the bath entangles with the quantum dot, and destroys the quantum correlations between the dot and the conduction electrons. When $\mathcal{V}$ is strong enough to make $\langle 0|\rho|1\rangle$ negligible, we have $\rho_f = a_1|0\rangle\langle 0| + a_2|1\rangle\langle 1|$ and, as shown above, the Kondo effect is destroyed. The measurement of the spin of the quantum dot represents complete measurement of the Kondo center (i.e., complete information about the Kondo center state is obtained). However, as we just saw above, a complete and an incomplete measurement have practically the same effect in reducing the resonance.

Destruction of the Kondo effect by a decohering action of an external microwave field has been considered in Ref. [12]. However, the external radiation works similar to increasing temperature, smearing the Kondo effect, and represents an effect of dissipation. In contrast to dissipation, the decoherence caused by the measurement leads to the pure decrease of the amplitude of the resonance (by a factor of $Z = 1 - \langle n_f \rangle$), without any smearing. The difference between decoherence and dissipation is one of the most important points in the modern theory of decoherence [10,11]. Decoherence is a loss of subtle phase relation between different states of the system, and is not necessarily associated with the flow of energy between the system and the bath, as exemplified, e.g., by Ref. [11]. In contrast, when the flow of energy between the system and the bath is present, both dissipation and decoherence take place.

Here, we consider decoherence by a spin bath [21], which allows a clear demonstration of the dissipationless suppression of the Kondo effect. Such a bath can be implemented in experiments by doping GaAs with manganese (so that the magnetic moments of Mn ions form the bath), iron or chromium impurities. The interaction between the spins of impurities is weak but not negligible: it determines the chaotic (or close to chaotic) dynamics of the bath, and our simulations show that this is a qualitatively important detail. Chaotic baths exhibit similar behavior for a wide range of parameters, while the results obtained for non-chaotic baths often depend on the specific details (initial conditions, form of the bath Hamiltonian, etc.) especially in the region of small J. We will discuss this in detail elsewhere.

Rigorous treatment of this problem is very difficult, but qualitative features of decoherence of a Kondo system can be studied by representing the spin state of the subsystem of conduction electrons by a single collective spin 1/2. Instead of the complete s–d exchange Hamiltonian describing the Kondo effect for spin degrees of freedom [2] we consider a qualitatively similar Hamiltonian

$$H = J\mathbf{S}_1\mathbf{S}_2 + \mathbf{S}_1 \sum_{j=1}^{N} A_j \mathbf{I}_j + H_B. \tag{8}$$

Here, $\mathbf{S}_1$ is the spin of the quantum dot ($S_1 = 1/2$), $\mathbf{S}_2$ represents the collective spin of the conduction electrons ($S_2 = 1/2$), $\mathbf{I}_j$ are the environmental spins ($I_j = 1/2$), and A_j are the coupling constants of the spin of the dot $\mathbf{S}_1$ with the bath spins. The Hamiltonian H_B governs the chaotic dynamics of the bath.

The Hamiltonian (8), although being extremely simplified in comparison with a realistic Kondo system, is suitable for qualitative description of the basic physics of the decoherence process. The essence of the Kondo effect is formation of a singlet ground state with an energy gain of order of T_K in comparison with the energy of the Fermi sea [1]. The lowest *triplet* state of the Kondo system corresponds to the non-interacting situation, when the conduction electrons just avoid the scattering center, and the energy of the lowest triplet state is just the Fermi sea energy. Thus, the energy separation between the lowest singlet and lowest triplet states, qualitatively described as J in Eq. (8), is of order of T_K. The most important difference between a real Kondo system and the model (8) is that the Kondo system has continua of the singlet states and triplet states. However, most of the states belonging to these continua are not important provided that all A_j are much smaller than the bandwidth of the conduction electrons. In this case, the interaction between the Kondo system and the bath is close to adiabatic, and only the low-lying singlet and triplet states are important.

Thus, we use the Hamiltonian (8) for qualitative analysis of the decoherence process of a Kondo system by a spin bath, with $J = E^* \sim T_K$. The entanglement between $\mathbf{S}_1$ and $\mathbf{S}_2$ can be described using the reduced density matrix $\rho = \mathrm{Tr}_{\{\mathbf{I}_j\}} W$, where W is the density matrix of the whole system (the two central spins $\mathbf{S}_{1,2}$ plus the all the bath spins), and the trace is taken over the bath spins $\{\mathbf{I}_j\}$. The entangled singlet state of the quantum dot and the conduction electrons (where the Kondo effect is maximal) corresponds to the non-diagonal element $\rho_{12} = \langle\uparrow\downarrow|\rho|\downarrow\uparrow\rangle = -1/2$. The decay of entanglement between $\mathbf{S}_1$ and $\mathbf{S}_2$ is characterized by a decrease of the absolute value of ρ_{12}. When ρ_{12} vanishes, the Kondo effect disappears.

We study this process by direct numerical solution [22] of the compound "system-plus-bath" time-dependent Schrödinger equation with the Hamiltonian (8). Initially, the system and the bath are in an uncorrelated product state. The initial state of the Kondo system is the singlet. The initial state of the bath is a random superposition of all basis states, which corresponds to low-temperature experiments when the temperature $J \gg T \gg A_j$. The chaotic dynamics of the bath has been simulated by using the form of H_B suggested in [23]; the level statistics has been verified to agree with the Wigner–Dyson distribution. The number of the bath spins has been varied from $N = 6$ to $N = 13$, and different sets of A_j have been used. The results of a sample run are shown in Fig. 1(a). Decoherence dynamics of the Kondo system and the decay of the element ρ_{12} are clearly seen.

The simulations illustrate the dynamics of the measurement process, which starts from the singlet state of the Kondo system. For the quantum dot with magnetic impurities, this dynamics is not relevant (since the initial product state is not realistic), but the final quasi-equilibrium state is absolutely meaningful, giving the correct value of ρ_{12}. This value, as well as general features of the system's evolution, is stable with respect to considerable changes in the parameters of H_B, number of spins, values of A_k, or variation of the initial conditions. Thus, the final state of the system represents the "pointer state" [11] which is robust with respect to decoherence.

The equilibrium value of ρ_{12} is determined by a competition between the exchange constant J and the strength of the system–bath interaction. Qualitative analysis suggests that the relevant quantity characterizing the system–bath coupling is the mean-square exchange $b = \sqrt{\sum_j A_j^2}$, so the final value of ρ_{12} is determined by the ratio J/b. Our results show that this is correct (see Fig. 1(b)): the results obtained with different number of the bath spins N, different sets of A_j, and different values of J, fall close to a universal curve $\rho_{12}(J/b)$. The scatter is moderate, stemming from the finite value of N and fluctuations present in the final state (Fig. 1(a)).

The suppression of the Kondo effect as a function of J/b is gradual, and the center of the transition corresponds to $J/b \approx 0.3$. For typical quantum dots [5], $J \sim T_K \approx 0.4$ K, and $A_k \sim I/n$ where $n \sim 10^7$ is the total number of the lattice sites inside the dot and $I \sim 1$ eV is the exchange of the impurity spin with the electron of the dot (the factor $1/n$ originates from the normalization of the wave function of the electron in the quantum dot). Thus, $b \sim \sqrt{n_i}\, I/n$ where n_i is the number of impurities in the dot. At large concentrations $x = n_i/n \sim 0.01$, Mn impurities in GaAs order ferromagnetically [24], and no longer form a spin bath, but for other ions, such as Fe [25], the critical concentration is larger, and $x \sim 0.01$ is acceptable. Therefore, in realistic systems

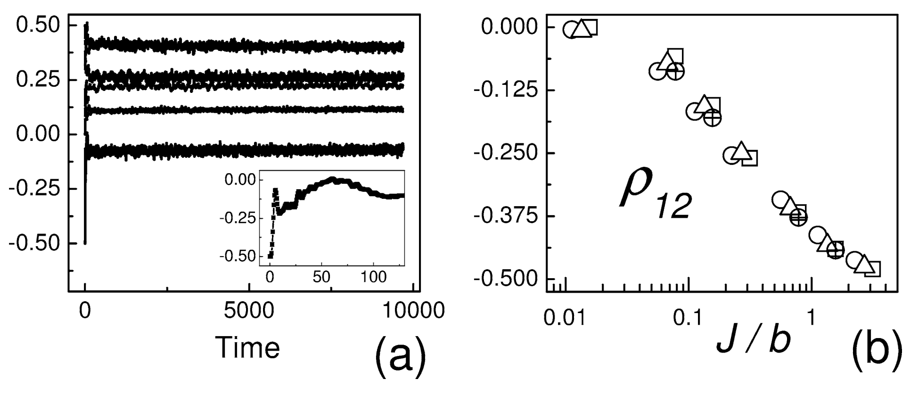

Fig. 1. Decoherence dynamics of the Kondo system by a spin bath. (a) Temporal evolution of the different elements of the density matrix ρ: diagonal elements corresponding to the states $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$ (four upper curves), and the non-diagonal element ρ_{12} (lowest curve). The inset shows initial decrease in ρ_{12}. (b) Dependence of the final value of ρ_{12} vs. the ratio J/b; the results for different N (from 6 to 13) and A_j are indicated by different symbols. Different results are close to the same universal curve.

$b \sim 0.1$–0.3 K, and the ratio $T_K/b \sim 0.3$ is, in principle, achievable, so that an experimental check of our predictions is possible. The experiment is rather straightforward: several Fe- or Cr-doped samples with different impurity concentration should be prepared, and the Kondo-anomaly should be measured, similar to [5]. The Kondo effect can be suppressed further by reducing the size of the dot (since the ratio J/b is proportional to n, i.e., to the volume of the dot). We also note that doping of the dot with non-magnetic impurities having internal degrees of freedom, e.g., with Ce atoms, will suppress the Kondo effect in exactly the same manner. Another possibility is to use a double quantum dot system [26], where the Kondo effect changes due to the presence of the "orbital" (right or left dot) degree of freedom, in addition to the spin of the dots. The measurement of the electron presence in a given dot [19] will bring the Kondo effect to the single dot regime.

Summarizing, we have shown that the quantum measurement of the spin in a Kondo system suppresses the Abrikosov–Suhl resonance and destroys the Kondo effect. This suppression is caused by the decohering influence of the measuring apparatus, and does not involve dissipation, i.e., it is qualitatively different from the dissipative suppression of the Kondo effect [12]. The effect predicted here can be studied in realistic experiments on quantum dots doped with magnetic (Fe, Cr) or non-magnetic (Ce) impurities, where the bath of impurities decoheres the Kondo system in the same way as a measuring device. The estimates suggest that such an experiment is already achievable with today's experimental techniques.

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References

[1] J. Kondo, Prog. Theor. Phys. 32 (1964) 37.
[2] A.C. Hewson, The Kondo Problem to Heavy Fermions, Cambridge Univ. Press, Cambridge, 1993.
[3] D.L. Cox, A. Zawadowski, Adv. Phys. 47 (1998) 599.
[4] L.I. Glazman, M.E. Raikh, JETP Lett. 47 (1988) 452; T.K. Ng, P.A. Lee, Phys. Rev. Lett. 61 (1988) 1768.
[5] D. Goldhaber-Gordon, H. Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav, M.A. Kastner, Nature 391 (1998) 156; S.M. Cronenwett, T.H. Oosterkamp, L.P. Kouwenhoven, Science 281 (1998) 540; J. Schmid, J. Weis, K. Eberl, K. von Klitzing, Physica B 256–258 (1998) 182; W.G. van der Wiel, S. De Franceschi, T. Fujisawa, J.M. Elzerman, S. Tarucha, L.P. Kouwenhoven, Science 289 (2000) 2105.
[6] Yu. Kagan, A.J. Leggett (Eds.), Quantum Tunneling in Condensed Media, North-Holland, Amsterdam, 1992, Section 2.
[7] V. Madhavan, W. Chen, T. Jamneala, M.F. Crommie, N.S. Wingreen, Science 280 (1998) 567; J. Li, W.-D. Schneider, R. Berndt, B. Delley, Phys. Rev. Lett. 80 (1998) 2893.
[8] H.C. Manoharan, C.P. Lutz, D.M. Eigler, Nature 403 (2000) 512.
[9] O.Yu. Kolesnichenko, R. De Kort, M.I. Katsnelson, A.I. Lichtenstein, H. van Kempen, Nature 415 (2002) 507.
[10] A. Peres, Quantum Theory: Concepts and Methods, Kluwer, Dordrecht, 1993.
[11] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, H.D. Zeh (Eds.), Decoherence and the Appearance of a Classical World in Quantum Theory, Springer, Berlin, 1996; W.H. Zurek, Phys. Rev. D 24 (1981) 1516; W.H. Zurek, Phys. Rev. D 26 (1982) 1862; E. Joos, H.D. Zeh, Z. Phys. B 59 (1985) 223.
[12] A. Kaminski, V.Yu. Nazarov, L.I. Glazman, Phys. Rev. Lett. 83 (1999) 384.
[13] O. Gunnarsson, K. Schönhammer, Phys. Rev. B 28 (1983) 4315.
[14] O. Újsághy, J. Kroha, L. Szunyogh, A. Zawadowski, Phys. Rev. Lett. 85 (2000) 2557.
[15] S.A. Gurvitz, Phys. Rev. Lett. 85 (2000) 812.
[16] M.I. Katsnelson, V.V. Dobrovitski, B.N. Harmon, Phys. Rev. A 62 (2000) 022118.
[17] M.I. Katsnelson, V.V. Dobrovitski, B.N. Harmon, Phys. Rev. B 63 (2001) 212404.
[18] J. von Neumann, Mathematical Foundations of Quantum Mechanics, Princeton Univ. Press, Princeton, 1955.
[19] B. Elattari, S.A. Gurvitz, Phys. Rev. Lett. 84 (2000) 2047.
[20] W. Lu, Z. Ji, L. Pfeiffer, K.W. West, A.J. Rimberg, Nature 423 (2003) 422;

A. Aasime, G. Johansson, G. Wendin, R.J. Schoelkopf, P. Delsing, Phys. Rev. Lett. 86 (2001) 3376;
E. Buks, R. Schuster, M. Heiblum, D. Mahalu, V. Umansky, Nature 391 (1998) 871;
H.J. Lynch, L. DiCarlo, L.I. Childress, et al., Bull. Am. Phys. Soc. 48 (2003) 1312.

[21] A. Garg, in: L. Gunther, B. Barbara (Eds.), Quantum Tunneling of Magnetization QTM'94, Kluwer, Dordrecht, 1995, p. 273;
N.V. Prokof'ev, P.C.E. Stamp, in: L. Gunther, B. Barbara (Eds.), Quantum Tunneling of Magnetization QTM'94, Kluwer, Dordrecht, 1995, p. 347;
N.V. Prokof'ev, P.C.E. Stamp, Rep. Prog. Phys. 63 (2000) 669.

[22] H.A. De Raedt, A. Hams, K. Michielsen, K. De Raedt, Comput. Phys. Commun. 132 (2000) 1.

[23] B. Georgeot, D.L. Shepelyansky, Phys. Rev. E 62 (2000) 6366.

[24] H. Ohno, Science 281 (1998) 951;
H. Ohno, J. Magn. Magn. Mater. 200 (1999) 110.

[25] Y.L. Soo, G. Kioseoglou, S. Huang, et al., Phys. Rev. B 63 (2001) 195209.

[26] U. Wilhelm, J. Weis, Physica E 6 (2000) 668;
T. Pohjola, H. Schoeller, G. Schoen, Europhys. Lett. 54 (2001) 241.