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Event-by-event simulation of EPR-Bohm experiments

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Summary. We present a computer simulation model that is strictly causal and local in Einstein's sense, does not rely on concepts of quantum theory but can nevertheless reproduce the results of quantum theory for the single-spin expectation values and two-spin correlations in an Einstein-Podolsky-Rosen-Bohm experiment.

1.1 Introduction

Computer simulation is a powerful methodology to model physical phenomena that is complementary to theory and experiment [1]. In this approach, we usually start from the basic equations of physics and employ numerical algorithms to solve these equations. But what if, as in quantum theory, the basic equation that describes the individual events is not known? Last year, also at this workshop, we discussed a simulation method that uses locally-connected networks of processing units with a primitive learning capability to generate events at a rate that agrees with the quantum mechanical probability distribution [2]. The fact that this simulation approach only uses causally local processes raises the question whether can also simulate Einstein-Podolsky-Rosen (EPR) [3] experiments and reproduce the results of quantum theory. This contribution demonstrates that the answer to this question is affirmative.

Quantum mechanical descriptions and experimental realizations of an EPR-Bohm gedanken experiment often adopt the example proposed by Bohm and Aharonov (EPRB) [4, 5]. This model, sketched in Fig. 1, considers a source that produces pairs of spin-1/2 particles, prepared in the singlet state $|\mathbf{\Psi}\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/2^{1/2}$. The two particles with opposite spins move in free space and in opposite directions. The spins of the individual particles are measured by means of Stern-Gerlach magnets. After passing a Stern-Gerlach magnet, the particle is detected at either detector $D_{+,i}$ or $D_{-,i}$, where $i = 1, 2$

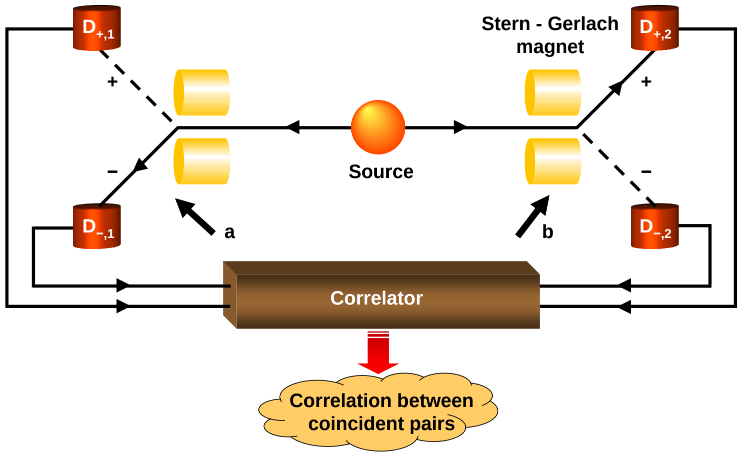

Fig. 1.1. Diagram of the Einstein-Podolsky-Rosen-Bohm experiment.

denotes the position of the pair of detectors with respect to the source (see Fig. 1). The firing of $D_{+,i}$ ($D_{-,i}$) defines the event at which we assign the value of spin up (+1) (spin down (−1)) to particle i. Representing the direction of magnet 1 (2) by the unit vector **a** (**b**), quantum mechanics yields [6] $\langle\mathbf{\Psi}|\sigma_\mathbf{1}\cdot\mathbf{a}|\mathbf{\Psi}\rangle = \langle\mathbf{\Psi}|\sigma_\mathbf{2}\cdot\mathbf{b}|\mathbf{\Psi}\rangle = 0$ and

$$\langle\mathbf{\Psi}|\sigma_\mathbf{1}\cdot\mathbf{a}\,\sigma_\mathbf{2}\cdot\mathbf{b}|\mathbf{\Psi}\rangle = -\mathbf{a}\cdot\mathbf{b}, \tag{1.1}$$

where $\sigma_i = (\sigma_i^x, \sigma_i^y, \sigma_i^z)$ are the three Pauli spin operators of particle $i = 1, 2$.

Experimentally, each Stern-Gerlach magnet measures the individual spins. Quantum theory itself has nothing to say about these individual assignments (quantum measurement paradox) [7]. The fundamental problem is to explain how individual events, recorded in space-time separated regions under conditions such that the measurement on one particle cannot have a causal effect on the result of the measurement on the other particle (Einstein's local causality criterion), exhibit the correlations Eq. (1.1). In this paper, we present a solution of this puzzle.

1.2 Analysis of a typical EPR-Bohm experiment

As a first step in solving this puzzle, it is necessary to determine the set of relevant data that is collected in a typical EPRB experiment. Apart from the signals generated by the detectors $D_{+,i}$ and $D_{-,i}$ ($i = 1, 2$), any experimental procedure that registers pairs of particles requires some criterion to decide whether two particles form a pair or not [8]. In EPRB experiments, this decision is taken on the basis of coincidence in time. Thus, a practical criterion for coincidence always involves a coincidence window (see Ref. [8, 9]). Therefore, the set of numbers, collected during one run of an EPRB experiment consists of two subsets (one subset for each observation station $i = 1, 2$). Each subset is a collection of triples [9]

$$\Upsilon_i = \{x_{n,i} = \pm 1, t_{n,i}, \mathbf{c}_{n,i} | n = 1, \ldots, N\} , \qquad (1.2)$$

where n labels the events, N is the total number of events in the run, $x_{n,i}$ tells us which of the two detectors at station i fired, $t_{n,i}$ holds the value of the time tag for event number n, and $\mathbf{c}_{n,i}$ denotes the direction of the magnets ($\mathbf{c}_{n,1} = \mathbf{a}_n$, $\mathbf{c}_{n,2} = \mathbf{b}_n$) when the nth pair of particles passes through the magnets.

After all data has been collected, the two subsets are analyzed for coincidences [9]. Coincidences are identified by calculating the time differences between the time tags of the different subsets and comparing these with a time window W (typically a few ns [9]). Denoting the number of coincidences between detectors $D_{x,1}$ ($x = \pm 1$) at station 1 and detectors $D_{y,2}$ ($y = \pm 1$) at station 2 by $C_{xy}(\mathbf{a}, \mathbf{b})$, we have

$$C_{xy}(\mathbf{a}, \mathbf{b}) = \sum_{n=1}^{N} \delta_{x,x_{n,1}} \delta_{y,x_{n,2}} \delta_{\mathbf{a},\mathbf{c}_{n,1}} \delta_{\mathbf{b},\mathbf{c}_{n,2}} \Theta(W - |t_{n,1} - t_{n,2}|), \qquad (1.3)$$

where $\Theta(t)$ is the Heaviside step function and we made a minor abuse of notation by representing the direction of the magnets by discrete labels (which is allowed because in experiment, the number of different directions is necessarily finite, hence representable by integer numbers). Note that the numerator of Eq. (1.3) is the number of all detected pairs. The correlation $E(\mathbf{a}, \mathbf{b})$ is given by [5, 6]

$$E(\mathbf{a}, \mathbf{b}) = \frac{C_{++}(\mathbf{a}, \mathbf{b}) + C_{--}(\mathbf{a}, \mathbf{b}) - C_{+-}(\mathbf{a}, \mathbf{b}) - C_{-+}(\mathbf{a}, \mathbf{b})}{C_{++}(\mathbf{a}, \mathbf{b}) + C_{--}(\mathbf{a}, \mathbf{b}) + C_{+-}(\mathbf{a}, \mathbf{b}) + C_{-+}(\mathbf{a}, \mathbf{b})} . \qquad (1.4)$$

The puzzle to be solved is how to generate the data set $\{\Upsilon_1, \Upsilon_2\}$ under the rather stringent condition that for all events $n = 1, \ldots, N$ and $i = 1, 2$:

$$x_{n,i} = f(\mathbf{c}_{n,i}, \mathbf{S}_{n,i}), \quad t_{n,i} = g(\mathbf{c}_{n,i}, \mathbf{S}_{n,i}), \qquad (1.5)$$

such that $E(\mathbf{a}, \mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}$. In Eq. (1.5), $\mathbf{S}_{n,i}$ represents the spin of the particle. The functions f and g in Eq. (1.5) obey Einstein's criterion of local causality: The values of the measured quantities at station 1 (2) are arithmetically independent of the choice of the settings at station 2 (1), for each individual particle generated by the source.

1.3 Computer simulation algorithm

Space limitations prevent us from discussing the motivation that has led us to the following algorithm:

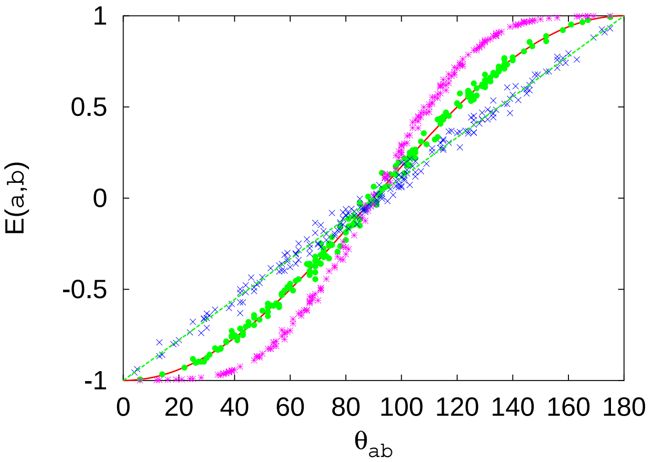

Fig. 1.2. Simulation results for the two-spin correlation for $W/T = \tau/T = 0.00025$, $N = 10^6$, and $M = 200$ randomly chosen values of $\mathbf{a}\cdot\mathbf{b} = \cos\theta_{\mathbf{ab}}$ covering the interval $[-1, +1]$. Crosses (blue): $d = 0$; Bullets (green): $d = 3$; Stars (pink): $d = 6$; Solid line (red): Quantum theory; Dashed line (green): $\theta_{\mathbf{ab}}/90 - 1$ (Bell-type model).

1. Specify the number of events N, the time-tag resolution τ/T (the actual value of the time scale T is irrelevant), the time window $W = k\tau$ ($k = 1, 2, \ldots$), and the number M of directions $\mathbf{a}$ and $\mathbf{b}$. Use random numbers to fill the arrays $(\mathbf{a}_1, \ldots, \mathbf{a}_M)$ and $(\mathbf{b}_1, \ldots, \mathbf{b}_M)$ with unit vectors. Set $n = 0$ and $C_{xy}(\mathbf{a}_m, \mathbf{b}_{m'}) = 0$ for all $x, y = \pm 1$ and $m, m' = 1, \ldots, M$.
2. While $n < N$, increment n by one and repeat steps 2 to 6.
3. Use uniform random numbers $-1 \le z_n \le 1$ and $0 \le \phi_n < 2\pi$ to assign the spin $\mathbf{S}_{n,1} = -\mathbf{S}_{n,2} = ((1 - z_n^2)^{1/2}\cos\phi_n, (1 - z_n^2)^{1/2}\sin\phi_n, z_n)$.
4. Use uniform random numbers $1 \le m, m' \le M$ to select the directions $\mathbf{a} = \mathbf{a}_m$ and $\mathbf{b} = \mathbf{b}_{m'}$.
5. The time tag $t_{n,1}$ ($t_{n,2}$) is obtained by generating a uniform random number in the interval $[0, T(1 - (\mathbf{S}_{n,1}\cdot\mathbf{a})^2)^{d/2}]$ ($[0, T(1 - (\mathbf{S}_{n,2}\cdot\mathbf{b})^2)^{d/2}]$) where d is a parameter of the simulation model. Compute $x = \mathrm{sign}(\mathbf{S}_{n,1}\cdot\mathbf{a})$ and $y = \mathrm{sign}(\mathbf{S}_{n,2}\cdot\mathbf{b})$.
6. Apply the coincidence criterion: If $|\lfloor t_{n,1}/\tau\rfloor - \lfloor t_{n,2}/\tau\rfloor| \le k$ the particles form a pair. Then, increment the count $C_{xy}(\mathbf{a}_m, \mathbf{b}_{m'})$. Go back to step 2.
7. After generating N events, we compute the correlations according to Eq. (1.4) and plot the results as a function of $\mathbf{a}\cdot\mathbf{b}$.

It is evident that this algorithm satisfies Einstein's criterion of local causality. The use of random numbers to select $\mathbf{S}_{n,i}$ and $t_{n,i}$ is not essential but convenient [10], and merely mimics the apparent unpredictability of the data.

Figure 1.2 shows simulation results for different values of the parameter d. For $d = 3$, our algorithm generates data that agrees with the quantum theory (solid line in Fig. 1.2). In fact, for $d = 3$ we can prove analytically that $\lim_{W\to 0}\lim_{N\to\infty}\lim_{\tau\to 0} E(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}$ [10]. For $d = 0$, the time-tag data is not used to determine the coincidences. Then, our model is a realization of the models studied by Bell, hence it cannot reproduce the correct quantum correlation Eq. (1.1) [5]. For $d > 3$, as illustrated by the data for $d = 6$ in Fig. 1.2, our simulation model produces correlations that are "stronger" than quantum correlation in the sense that $|E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}')| + |E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')|$ can exceed the quantum limit $2\sqrt{2}$. To simulate experiments that use the photon polarization [9, 11], we replace the three-component spin $\mathbf{S}_{n,1}$ by a two-component spin [10]. For $d = 2$, we find (results not shown) that the simulation reproduces the results of quantum theory, a fact that we can also prove analytically [10].

Summarizing: Starting from nothing more than the observation that an EPRB experiment produces the set of data $\{\Upsilon_1, \Upsilon_2\}$, we have constructed event-based computer simulation models that do not rely on concepts of quantum theory but reproduce the correlation Eq. (1.1) that is characteristic for a quantum system in the most entangled state.

References

1. D.P. Landau and K. Binder, *A Guide to Monte Carlo Simulation in Statistical Physics*, Cambridge University Press, Cambridge (2000).
2. H. De Raedt, K. De Raedt, K. Michielsen, and S. Miyashita, Comp. Phys. Comm. (in press); K. De Raedt, H. De Raedt, and K. Michielsen, Comp. Phys. Comm. 171, 19 (2005); K. Michielsen, K. De Raedt, and H. De Raedt, J. Comp. Theor. Nanoscience 2, 227 (2005); H. De Raedt, K. De Raedt, and K. Michielsen, J. Phys. Soc. Jpn. Suppl. 76, 16 (2005); H. De Raedt, K. De Raedt, and K. Michielsen, Europhys. Lett. 69, 861 (2005).
3. A. Einstein, A. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).
4. D. Bohm and Y. Aharonov, Phys. Rev. 108, 1070 (1957).
5. J.S. Bell, *Speakable and unspeakable in quantum mechanics*, Cambridge University Press, Cambridge UK (2003).
6. L.E. Ballentine, *Quantum Mechanics: A Modern Development*, World Scientific, Singapore (2003).
7. D. Home, *Conceptual Foundations of Quantum Physics*, Plenum Press, New York (1997).
8. J.F. Clauser and M.A. Horne, Phys. Rev. D 10, 526 (1974).
9. G. Weihs, T. Jennewein, C. Simon, H. Weinfurther, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998).
10. K. De Raedt et al, to be published.
11. A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. 49, 91 (1982); Phys. Rev. Lett. 49, 1804 (1982); W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, Phys. Rev. Lett. 81, 3563 (1998).