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Event-by-event simulation of Wheeler's delayed-choice experiment

S. Zhao[a], S. Yuan[a], H. De Raedt[a], K. Michielsen[b]

[a]Department of Applied Physics, Zernike Institute for Advanced Materials
University of Groningen, Nijenborgh 4, NL-9747 AG, Groningen, The Netherlands
[b]EMBD, Vlasakker 21, B-2160 Wommelgem, Belgium

Abstract

We present a computer simulation model of Wheeler's delayed choice experiment. The model is solely based on experimental facts and does not rely on concepts of quantum theory or probability theory. We demonstrate that it is possible to give a particle-only description of Wheeler's delayed choice experiment which reproduces the averages calculated from quantum theory and which does not defy common sense. ⓒ 2010 Published by Elsevier Ltd.

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1. Introduction

According to the wave-particle duality, a concept of quantum theory, photons exhibit both wave and particle behavior depending upon the circumstances of the experiment [1]. In 1978, Wheeler proposed a gedanken experiment [2], a variation on Young's double slit experiment, in which the decision to observe wave or particle behavior is postponed until the photon has passed the slits.

An almost ideal experimental realization of Wheeler's delayed choice experiment has been reported recently [3]. The experimental set-up (Fig. 1(left)) consists of a single-photon source, a Mach-Zehnder interferometer, with at the output side a beam splitter (BS$_{output}$), the presence of which can be controlled by a voltage applied to an electro-optic modulator (EOM) and detectors. The key point in this experiment is that the decision to apply a voltage to the EOM is made after the photon has passed BS$_{input}$. Logically, this experiment is equivalent to the one shown in Fig. 1(right). An essential feature of this experiment is that the experimenter can decide, at any time, whether or not BS$_{output}$ is present. In Fig. 1(right), we symbolize this by saying that the presence/absence of BS$_{output}$ is controlled by a binary (pseudo) random number A_n.

Although the detection events are the only experimental facts, according to Ref. [3], the pictorial description of what transpires in the experiment is as follows: If BS$_{output}$ is absent, then the arrival of a photon at either detector clearly gives which-way information about the photon within the interferometer (particle behavior), with 50% arriving from either path. If BS$_{output}$ is present, the paths interfere and it is impossible to know which path the photon took (wave behavior). Accordingly, the detectors register an interference pattern.

The outcome of the experiment, that is the averages over many detection events, are in agreement with quantum theory [3]. However, the pictorial description, as given in Ref. [3], defies common sense: If the decision to leave in place or take away BS$_{output}$ is made after the photon left BS$_{input}$ but before it passes BS$_{output}$, this decision influences the behavior of the photon in the past and changes the representation of the photon from a wave to a particle [3].

On the other hand, the pictorial description (which is commonly adopted in discussions of Wheeler's delayed choice experiment) uses concepts from quantum theory, a theory that cannot describe single events [1]; it provides a
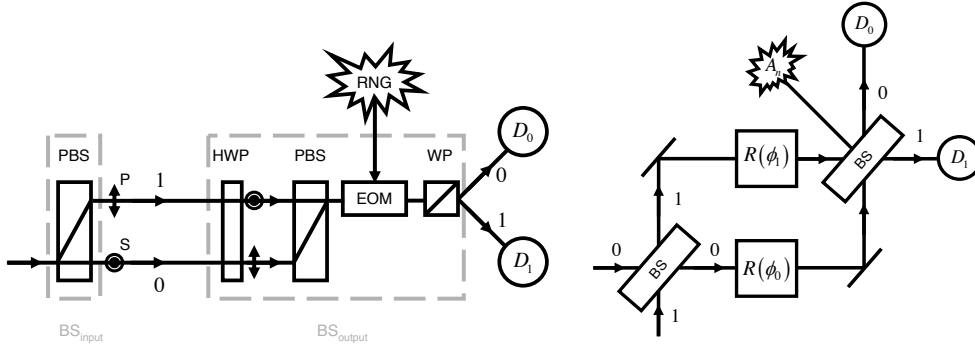
Figure 1: Left: Schematic diagram of the experimental setup for Wheeler's delayed-choice gedanken experiment [3]. PBS: Polarizing beam splitter; HWP: Half-wave plate; EOM: electro-optic modulator; RNG: Random number generator; WP: Wollaston prism; P,S: Polarization state of the photon; D_0, D_1: Detectors. Right: Logically equivalent, simplified representation of the ideal Wheeler's delayed choice gedanken experiment. The network consists of two beam splitters (BS), two devices $R(\phi_0)$ and $R(\phi_1)$ that induce a phase shift $\phi_0 - \phi_1$ between the two routes 0 and 1, two perfectly reflecting mirrors and two detectors D_0 and D_1. A (pseudo) random number A_n determines whether or not the second BS should be physically taken away when the nth photon has passed the first BS.

recipe to compute averages only. Therefore, it should not be a surprise that the application of concepts of quantum theory to the domain of individual events may lead to conclusions that are at odds with common sense.

2. Simulation model

The model presented in this paper builds on earlier work [4, 5, 6, 7, 8, 9, 10, 11, 12] in which we have demonstrated that it may be possible to simulate quantum phenomena on the level of individual events without invoking a single concept of quantum theory or probability theory.

In our simulation approach, a messenger (representing a photon), carries a message (representing the phase of the photon) and is routed through the network and the various units that process the messages. We now explicitly describe our simulation model that is, we specify the message carried by the messengers, the algorithms that simulate the processing units and the data analysis procedure.

Messenger. Particles carry a message represented by a two-dimensional unit vector $\mathbf{y}_{k,n} = (\cos\psi_{k,n}, \sin\psi_{k,n})$ where $\psi_{k,n}$ refers to the phase of the photon. The subscript $n \geq 0$ numbers the consecutive messages and $k = 0, 1$ labels the channel of the beam splitter at which the message arrives.

Beam splitter. The event-by-event processing of the single-photon beam splitters is modeled by the DLM-based processor depicted in Fig. (2)(feft), where DLM stands for deterministic learning machine [4, 5]. The DLM-based processor consists of an input stage (DLM), a transformation stage (T), an output stage (O) and has two input and two output channels, labeled with $k = 0, 1$.

The input stage receives a message on either input channel 0 or 1, never on both channels simultaneously. The arrival of a message on channel 0 (1) corresponds to an event of type 0 (1). The input events are represented by the vectors $\mathbf{v}_n = (1, 0)$ or $\mathbf{v}_n = (0, 1)$ if the n -th event occurred on channel 0 or 1, respectively. The DLM has two internal registers $\mathbf{y}_{k,n} = (C_{k,n}, S_{k,n})$ and one internal vector $\mathbf{x}_n = (x_{0,n}, x_{1,n})$, where $x_{0,n} + x_{1,n} = 1$ and $x_{i,n} > 0$. These three two-dimensional vectors are labeled by the message number n because their content is updated every time the DLM receives a message. Before the simulation starts we set $\mathbf{x}_0 = (x_{0,0}, x_{1,0}) = (r, 1 - r)$, where r is a uniform pseudo-random number. In a similar way we use pseudo-random numbers to set $\mathbf{y}_{0,0}$ and $\mathbf{y}_{1,0}$. Upon receiving the $(n + 1)$th input event, the DLM performs the following steps: It stores the message $\mathbf{y}_{k,n+1} = (\cos\psi_{k,n+1}, \sin\psi_{k,n+1})$ in its internal register $\mathbf{y}_{k,n+1} = (C_{k,n+1}, S_{k,n+1})$; It updates its internal vector according to the rule

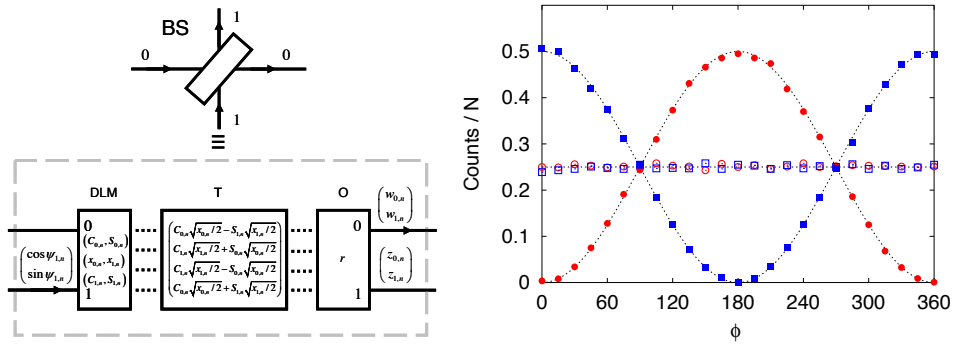$$x_{i,n+1} = \alpha x_{i,n} + (1 - \alpha)\delta_{i,k}, \tag{1}$$

Figure 2: Left: Diagram of a DLM that performs an event-based simulation of a single-photon beam splitter (BS). The solid lines represent the input and output channels of the BS. The presence of a message is indicated by an arrow on the corresponding channel line. The dashed lines indicate the data flow within the BS. Right: Simulation results for the diagram shown in Fig. 1(right). Input channel 0 receives $(\cos\psi_0, \sin\psi_0) = (1, 0)$. Input channel 1 receives no events. Initially, the rotation angles $\phi_0 = \phi_1 = 0$ and after each set of $N = 10000$ events, ϕ_0 is increased by 15 degrees while $\phi_1 = 0$. For each input event, a pseudo-random number A_n is used to open or close the interferometer configuration. Open (closed) markers give the simulation results for the open (closed) configuration of the interferometer. Squares and circles give the simulation results for the normalized intensities N_0/N and N_1/N as a function of the phase shift $\phi = \phi_0 - \phi_1$. Lines represent the results of quantum theory. Simulations are carried out with $\alpha = 0.99$.

where $0 < \alpha < 1$ is a parameter that controls the learning process. By construction $x_{0,n+1} + x_{1,n+1} = 1$ and $x_{i,n+1} \geq 0$.

The transformation stage T takes as input the data stored in the two internal registers $\mathbf{y}_{k,n+1} = (\cos\psi_{k,n+1}, \sin\psi_{k,n+1})$ and in the internal vector $\mathbf{x}_{n+1} = (x_{0,n+1}, x_{1,n+1})$ and constructs the four-dimensional vector

$$\mathbf{T} = \frac{1}{\sqrt{2}}\begin{pmatrix} C_{0,n+1}\sqrt{x_{0,n+1}} - S_{1,n+1}\sqrt{x_{1,n+1}} \\ C_{1,n+1}\sqrt{x_{1,n+1}} + S_{0,n+1}\sqrt{x_{0,n+1}} \\ C_{1,n+1}\sqrt{x_{1,n+1}} - S_{0,n+1}\sqrt{x_{0,n+1}} \\ C_{0,n+1}\sqrt{x_{0,n+1}} + S_{1,n+1}\sqrt{x_{1,n+1}} \end{pmatrix}. \tag{2}$$

Rewriting this vector as a two-dimensional vector with complex-valued entries, it is easy to show that $\mathbf{T}$ corresponds to the matrix-vector multiplication in the quantum theoretical description of a beam splitter [1, 4, 5], namely

$$\begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} a_0 + ia_1 \\ a_1 + ia_0 \end{pmatrix}. \tag{3}$$

where (a_0, a_1) and (b_0, b_1) denote the input and output amplitudes of the photons in the 0 and 1 channels of a beam splitter.

If $w_{0,n+1}^2 + w_{1,n+1}^2 > r$ where $0 < r < 1$ is a uniform pseudo-random number, the output stage O sends the message

$$\mathbf{w}_{n+1} = (w_{0,n+1} + w_{1,n+1})/(w_{0,n+1}^2 + w_{1,n+1}^2)^{1/2}, \tag{4}$$

with $w_{0,n+1} = C_{0,n+1}x_{0,n+1}^{1/2} - S_{1,n+1}x_{1,n+1}^{1/2}$ and $w_{1,n+1} = C_{1,n+1}x_{1,n+1}^{1/2} + S_{0,n+1}x_{0,n+1}^{1/2}$ through output channel 0. Otherwise, output stage O sends the message

$$\mathbf{z}_{n+1} = (z_{0,n+1}, z_{1,n+1})/(z_{0,n+1}^2 + z_{1,n+1}^2)^{1/2}, \tag{5}$$

with $z_{0,n+1} = C_{1,n+1}x_{1,n+1}^{1/2} - S_{0,n+1}x_{0,n+1}^{1/2}$ and $z_{1,n+1} = C_{0,n+1}x_{0,n+1}^{1/2} + S_{1,n+1}x_{1,n+1}^{1/2}$ through output channel 1.

Phase shifters $R(\phi_0)$ and $R(\phi_1)$. These devices perform a plane rotation on the vectors (messages) carried by the particles. As a result the phase of the particles is changed by ϕ_0 or ϕ_1 depending on the route followed.

Detection and data analysis procedure. Detector D_0 (D_1) registers the output events at channel 0 (1). For fixed $\phi = \phi_1 - \phi_0$, a simulation run of N events generates the data set $\Gamma(\phi) = \{x_n, A_n | n = 1, \ldots, N\}$. Here $x_n = 0, 1$ indicates

which detector fired (D_0 or D_1), and $A_n = 0, 1$ tells us whether the second beam splitter, BS$_{output}$ was absent or not. Given the data set $\Gamma(\phi)$, we can easily compute the number of 0 (1) output events N_2 (N_3) for the open and closed configuration of the interferometer, from which we directly obtain the required averages.

3. Results

It is a simple, straightforward excercise to program the algorithm described earlier. Representative results of an event-by-event simulation of Wheeler's delayed choice experiment are shown in Fig. 2(right). Obviously, the simulation data are in quantitative agreement with those of quantum theory (and in qualitative agreement with experiment [3]). Elsewhere, we present the results of similar simulations that take into account the polarization [13], that is we simulate the real experiment [3]. Our simulations prove that it is possible to give a particle-only description for Wheeler's delayed choice experiment that reproduces the averages calculated from quantum theory and does not contradict common sense.

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