A modified Mach-Zehnder experiment to test the applicability of quantum theory to single-particle experiments

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ABSTRACT

We propose a modified single-particle Mach-Zehnder interferometer experiment in which the path length of one arm may change (randomly or systematically) according to the value of an external two-valued variable x, for each passage of a particle through the interferometer. Quantum theory predicts an interference pattern that is independent of the sequence of the values of x. On the other hand, corpuscular models that reproduce the results of quantum optics experiments carried out up to this date show a reduced visibility and a shift of the interference pattern depending on the details of the sequence of the values of x. The key question to be answered in a real laboratory experiment is: Which interference pattern is observed? Despite the general believe that quantum theory might be used to describe all single particle experiments, this is an interesting question to be answered since in the proposed experiment the experimental conditions not only continuously change but they might also have causal effects on the passage of the photons through the interferometer. The proposed experiment can be used to determine to what extent quantum theory provides a description of observed events beyond the usual statistical level.

Keywords: Interference, quantum theory, Mach-Zehnder interferometer, single photons, event-by-event simulation

1. INTRODUCTION

Particle-wave duality, a concept of quantum theory, attributes to photons the properties of both wave and particle behavior depending upon the circumstances of the experiment.[1] Identifying a click of a detector with the arrival of a particle, the particle nature of photons shows up in an experiment with of a single 50/50 beam splitter, of which only one input port is used, and a source emitting single photons and pairs of photons.[2] A key feature in their experiment is the use of the three-level cascade photon emission of the calcium atom. When the calcium atoms are excited to the third lowest level, they relax to the second lowest state, emitting photons of frequency f, followed by another transition to the ground state level causing photons of frequency f' to be emitted.[3] It is observed that each such two-step process emits two photons in two spatially well-separated directions, allowing for the cascade emission to be detected using a time-coincidence technique.[3] One of two light beams produced by the cascade is directed to a detector D. The other beam is sent through a 50-50 beam splitter to detectors D_0 and D_1. Time-coincidence logic is used to establish the emission of the photons by the three level cascade process: Only if detectors D and D_0, D and D_1, or D_0 and D_1 fire, a cascade emission event occurred. Then, the absence of a coincidence between the firing of detectors D_0 and D_1 is taken as unambiguous evidence that the photon created in the cascade and passing through the beam splitter behaves as one indivisible entity taking

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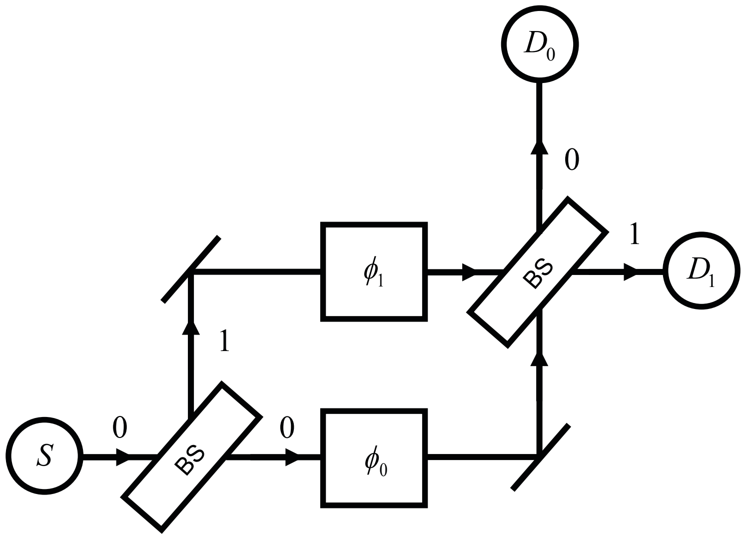

Figure 1. Schematic diagram of the Mach-Zehnder interferometer (MZI) experiment. S: Light source; BS: 50-50 beam splitter; ϕ_0: Phase shifter lower arm; ϕ_1: Phase shifter upper arm; D, D_0, D_1: Detectors.

only one of the two paths behind the beam splitter to the detector. The analysis of the experimental data favors the hypothesis that the photons created by the cascade process in the calcium atom are to be regarded as indivisible.[4] Therefore one could conclude that in this experiment the photon behaves as a particle.

Having established the corpuscular nature of single photons, Grangier *et al.*[2] extended the experiment by sending the photons emerging from the 50-50 beam splitter to another 50-50 beam splitter, thereby constructing a Mach-Zehnder interferometer (MZI), see Fig. 1 for a schematic picture of the experimental set-up. Note that the mirrors in the MZI do not alter the indivisible character of the photons. Grangier *et al.*[2] observed that after collecting many photons one-by-one, the normalized frequency distributions of detection counts recorded by the detectors D_0 and D_1 fit nicely to the interference patterns

$$I_0 = \sin^2 \frac{\phi_0 - \phi_1}{2}, \quad (1)$$

$$I_1 = \cos^2 \frac{\phi_0 - \phi_1}{2}. \quad (2)$$

Hence, the signal on the detectors D_0 and D_1 is modulated by the phase difference $\phi_0 - \phi_1$ between the two interferometer arms. The resulting interference pattern is the same as if the source would have emitted a "classical" monochromatic wave. Given the absence (within experimental accuracy) of a coincidence between the firing of detectors D_0 and D_1 one could conclude that the photons are indivisible particles that one-by-one build up an interference pattern, the latter often being associated with wave character.

The input to both experiments seem to be single photons, but the question arises how to interpret the final output which seems to show particle or wave character depending on the circumstances of the experiment. This question is not limited to photons. Already in 1924, de Broglie introduced the idea that also matter can exhibit wave-like properties.[5] This idea has been confirmed in various double-slit experiments with massive objects such as electrons,[6–9] neutrons,[10,11] atoms[12,13] and molecules such as C_{60} and C_{70},[14,15] all showing interference.

In the first experiment, although the average results after many detection events can be described by classical wave theory ($I_0 = I_1 = 0.5$), one can obtain full which-path information of the incoming photons, a property associated to particle behavior. Hence, one could give a complete description of the first experiment in terms of particles. In the MZI experiment, the average results after many detection events can also be described by classical wave theory, but now one observes interference fringes, associated to wave-like behavior, and no full which-path information of the incoming single photons can be obtained in the experiments. To resolve this apparent contradiction in the behavior of the photons, quantum theory introduces the concept of particle-wave duality.[1] Therefore explanations of this type of experiments are frequently given in terms of single photons and particle-wave duality.

However, the pictorial description using concepts from quantum theory, when applied to individual detection events (instead of to averages) leads to conclusions that defy common sense: The photon seems to change its representation from a particle to a wave while traveling from the source to the detector in the MZI experiment. This should not be a surprise: It is commonly accepted that quantum theory gives us a recipe to compute the frequency (averages) for observing events, but does not describe individual events.[1] Unfortunately, neither classical wave theory (Maxwell's theory), neither Newtonian mechanics (particle's equation of motion), nor does quantum theory give a single clue as how to explain the fact that individual detection events (non-coincident discrete detector clicks) are observed experimentally and, when collected over a sufficiently long time, yield averages (including interference patterns) that agree with wave theory. Since no theory seems to exist that can give a sensical description of the "whole" experiment, including the intermediate and final outcome, we adopted the idea to search for algorithms that could mimic (simulate) the detection events and experimental processes. In other words, the idea is to start from the observable facts that in both experiments

- the experimental set-up is an arrangement of certain optical apparatuses (single-photon source, beam-splitter, mirror, detector),
- the source is emitting single photons,
- the detectors D_0 and D_1 produce non-coincident discrete clicks,
- after many detector clicks have been registered, their averages can be described by wave theory,

and search for a set of rules (which cannot be obtained from experiment) for the photons and optical apparatuses that result in the same averages as obtained from the experiment.

In Refs.[16–18] we have proposed an event-based corpuscular model, see section 4 for a short description, which has shown to reproduce the statistical predictions of quantum theory for the single beam splitter and the MZI experiment of Grangier *et al.*[2] In a pictorial description of the simulation model, we may speak about "photons" generating the detection events. However, these so-called photons are elements of a model or theory for the real laboratory experiment only. The experimental facts are the settings of the various optical apparatuses and the detection events. What happens in between activating the source and the registration of the detection events is not measured and is therefore not known. Although in the event-based model one always has full which-path-information of the individual photons (one can always track the photons during the simulation), the photons build up an interference pattern at the detector. Hence, although, the appearance of an interference pattern is commonly considered to be characteristic for a wave, we have demonstrated that, as in experiment, it can also be build up by many photons.[16–18] Thus, in contrast to the quantum theoretical description of the MZI experiment in terms of averages over many events, the event-based corpuscular model provides a rational, logically consistent explanation of the experimental facts in terms of causal processes that are formulated as discrete events to which one can associate "particles".

Using the same algorithmic approach for modeling the single beam splitter and MZI experiment with single photons of Grangier *et al.*[2] (see Refs.[16–18]), we also modeled Wheeler's delayed choice experiment with single photons of Jacques *et al.*[19] (see Refs.[18,20,21]), the quantum eraser experiment of Schwindt *et al.*[22] (see Ref.[18,23]), double-slit and two-beam single-photon interference experiments and the single-photon interference experiment with a Fresnel biprism of Jacques *et al.*[24] (see Ref.[18,25]), quantum cryptography protocols (see Ref.[26]), the Hanbury Brown-Twiss experiment of Agafonov *et al.*[27] (see Ref.[18,28]), universal quantum computation (see Ref.[29,30]), the violation of Bell's inequalities in Einstein-Podolsky-Rosen-Bohm-type of experiments, involving two photons in the singlet state, of Aspect *et al.*[31,32] and Weihs *et al.*[33] (see Refs.[18,34–39]), and the propagation of electromagnetic plane waves through homogeneous thin films and stratified media (see Ref.[18,40]). A review of the simulation method and its applications is given in Ref.[18]

A crucial property of the event-based corpuscular models is that they reproduce "wave results" observed in different experiments without any change to algorithms modeling the photons and optical apparatuses.[18] These

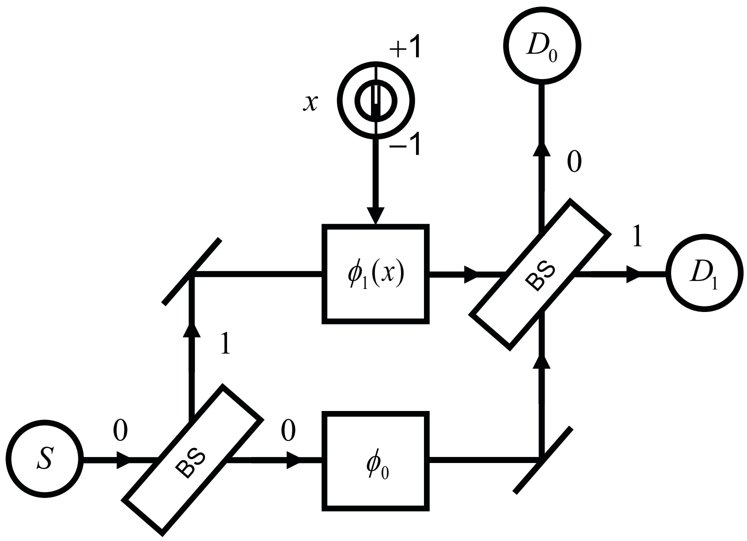

Figure 2. Schematic diagram of the modified MZI experiment. S: Light source; BS: 50-50 beam splitter; ϕ_0: Phase shifter lower arm; $\phi_1(x)$: Phase-shifter upper arm controlled by the external variable x; D, D_0, D_1: Detectors. In single photon experiments x may change before the photon enters the MZI but not during the passage of the photon through the MZI. For simplicity we consider experiments in which x takes the values -1 and $+1$ only. The recorded dataset for N detection events is given by $\{x_i, d_{0,i}, d_{1,i}, d_i | i = 1, \ldots N\}$ where $d_{k,i} = 1$ if detector D_k, $k = 0, 1$ fired and $d_{k,i} = 0$ otherwise, and $d_i = 1$ ($d_i = 0$) if detector D fired (did not fire). Note that the value of the experimental setting parameter x is known and certain at each moment in time.

algorithms can, of course, be simplified for particular experiments. For example, if photon polarization is not essential to a given experiment, then for simplicity we can omit the photon polarization in the event-based corpuscular model of this particular experiment.

Although these algorithms can be given an interpretation as a realistic cause-and-effect description that is free of logical difficulties, it is at present impossible to decide whether or not such algorithms are realized by Nature: Only new, dedicated experiments such as the one proposed in this paper can teach us more about this intriguing question.

Given the fact that the frequency distributions produced by the event-based corpuscular models cannot be distinguished from those predicted by quantum theory for the single-photon experiments performed so far and given the general belief that quantum theory can be used to describe all single-particle experiments, the key question is whether an experiment can be performed that shows a difference between the results obtained by quantum theory and those obtained by the event-based corpuscular model for this experiment. In this paper we propose a modified MZI experiment for which under particular experimental conditions quantum theory and the event-based corpuscular model predict a different outcome. Since one obviously cannot refute a model on the basis of quantum theoretical predictions alone, the question to be answered by a real laboratory experiment is: Which interference pattern is produced?

2. MODIFIED MACH-ZEHNDER INTERFEROMETER EXPERIMENT

Consider the modified MZI experiment (Fig. 2) in which the length of the upper arm can be varied by a control parameter x. According to Maxwell's theory, carrying out the experiment with a fixed value of x and with a coherent monochromatic light source S gives for the normalized intensities I_0 and I_1, recorded by the detectors D_0 and D_1,[41]

$$I_0 = \sin^2 \frac{\phi_0 - \phi_1(x)}{2}, \tag{3}$$

$$I_1 = \cos^2 \frac{\phi_0 - \phi_1(x)}{2}, \tag{4}$$

showing that the signal on the detectors D_0 and D_1, respectively, is modulated by the phase difference $\phi_0 - \phi_1(x)$.

In this single-photon experiment we allow the variable x to change before the photon enters the MZI but not during the passage of the photon through the MZI. For simplicity, but not out of necessity, we only consider experiments in which x takes the values +1 and -1 and for which $\phi_1(x=+1)\mathrm{mod}2\pi = 0$ and $\phi_1(x=-1)\mathrm{mod}2\pi = -\pi/2$. We consider a systematic and a random procedure to change x such that $x=+1$ and $x=-1$ occur with the same frequency. In the systematic procedure we replace x by $-x$ after the single photon source has emitted K photons. For $K=1$ this procedure leads to an alternating sequence of x-values. In the random procedure we use a random number to decide whether or not we replace x by $-x$ after the single photon source has emitted K photons. In both procedures we repeat this sequence such that the total number of photons emitted by the source equals N. Each click of the detector D_0 or D_1 is labeled by the currently known and certain value of x. After the N photons have been sent and all clicks have been registered, we count the number of detection events on D_0 and D_1 for each value of x separately, yielding the numbers $N_0(x)$ and $N_1(x)$. Finally, we define the normalized frequencies to detect photons by $F_0(x) = N_0(x)/(N_0(x)+N_1(x))$ and $F_1(x) = N_1(x)/(N_0(x)+N_1(x))$.

3. QUANTUM THEORY

According to wave theory,[41] the amplitudes $(b_0(x), b_1(x))$ of the photons in the output modes 0 and 1 of the MZI with a fixed value of x are given by

$$\begin{pmatrix} b_0(x) \\ b_1(x) \end{pmatrix} = ie^{i\varphi'(x)} \begin{pmatrix} \sin\varphi(x) & \cos\varphi(x) \\ \cos\varphi(x) & -\sin\varphi(x) \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}, \tag{5}$$

where the amplitudes of the photons in the input modes 0 or 1 are represented by a_0 and a_1, $\varphi(x) = (\phi_0 - \phi_1(x))/2$ and $\varphi'(x) = (\phi_0 + \phi_1(x))/2$. For the case at hand $a_1 = 0$ and without loss of generality, we may take $a_0 = 1$.

The Copenhagen interpretation maintains that the wave function provides a complete and exhaustive description of the experiment with an individual particle.[1,42] Therefore, grouping all detection events of the individual photons according to the corresponding values of x at the time of their passage through the MZI, the Copenhagen interpretation predicts that the probability distributions to register detection events at D_0 are given by

$$I_0(x=+1) = |b_0(x=+1)|^2 = \frac{1}{2}\sin^2\frac{\phi_0}{2}, \tag{6}$$

$$I_0(x=-1) = |b_0(x=-1)|^2 = \frac{1}{2}\sin^2\frac{\phi_0+\pi/2}{2}, \tag{7}$$

where the prefactor 1/2 comes from the fact that we have assumed that $x=+1$ and $x=-1$ occur with the same frequency. Note that Eqs. (6) and (7) are independent of the procedure that changes x.

If the detection events are not grouped according to the values of x, the Copenhagen interpretation predicts

$$I_0' = \frac{1}{2}\sin^2\frac{\phi_0}{2} + \frac{1}{2}\sin^2\frac{\phi_0+\pi/2}{2}. \tag{8}$$

Here and in the following the prime indicates that the detection events are not grouped (associated) with the current value of x at the time of detection.

Finally, if x does not change during the experiment

$$I_0''(x=+1) = \sin^2\frac{\phi_0}{2}, \tag{9}$$

$$I_0''(x=-1) = \sin^2\frac{\phi_0+\pi/2}{2}, \tag{10}$$

where the double prime indicates that the value of x is fixed during the experiment.

If quantum theory correctly describes the experiment with varying but always known x, we expect to find for the observed frequencies at detector D_0

$$F_0(x=+1) \approx I_0(x=+1) = \frac{1}{2}\sin^2\frac{\phi_0}{2}, \tag{11}$$

$$F_0(x=-1) \approx I_0(x=-1) = \frac{1}{2}\sin^2\frac{\phi_0+\pi/2}{2}, \tag{12}$$

(see Eqs. (6) and (7)) independent of the procedure for changing x being systematic or random and independent of the number of emitted photons K per change of x. In fact, quantum theory predicts that the result is completely independent of the sequence of x. Note that this cannot be true in general: One could consider $x = +1, \ldots, +1, -1$ so that there is only one event for $x = -1$. In this case the observed frequency does not correspond to an interference pattern although quantum theory predicts that also for this case $I_0(x=-1) = \sin^2(\phi_0+\pi/2)/2$.

It is precisely this feature, the fact that quantum theory predicts results that are independent of the sequence of x-values, that we propose to test experimentally. Note that there is no indication, let alone a kind of proof that quantum theory, being a theory that makes predictions about statistics only, correctly describes experiments in which the procedure for preparing the state of the photon (i.e. the state before the photon is being detected) can change with each photon.

4. EVENT-BASED CORPUSCULAR MODEL

Although detailed accounts of the event-based corpuscular modeling approach, with applications to many different single-photon experiments have been published elsewhere,[16–18, 20, 21, 23, 25, 26, 28–30, 34–39] for the reader's convenience, we briefly describe the simulation technique. The basic ideas of the simulation approach are that (i) we stick to what we know about the experiment, that is we consider the experimental configuration and its outcome as input for constructing the simulation algorithm; (ii) we try to invent a procedure, algorithm or set of rules that generates the same type of data as in experiment and reproduces the averages predicted by quantum theory; (iii) we keep compatibility with macroscopic concepts.

Generally speaking, the event-based corpuscular simulation method can be viewed as a message passing and message processing method in which the photons play the role of the messengers and the optical apparatuses, such as a (polarizing) beam splitter, polarizer, wave plate, detector and so on play the role of the processors that interpret and manipulate the messages. In what follows we briefly describe how we model the photon and the optical apparatuses that are sufficient to simulate a MZI experiment. This means that here we do not consider the polarization of the photon and that we consider detectors that simply count the detection events. More sophisticated models for the photon and the detectors can be found in Refs.[18, 20, 21, 23, 26, 34–37, 39] and Refs.,[18, 25, 28] respectively. Note that these more sophisticated event-based corpuscular models have also been used to simulate the MZI experiment. They would, however, unnecessarily complicate the modeling and pictorial description of the experiments we consider here. To simulate the MZI experiment we make use of the following models:

- Photon: We consider the photon to be a particle having an internal clock with one hand that rotates with a frequency $f = \omega/2\pi$. Hence, the rotation velocity of the hand depends on the angular frequency ω, that is the "color" of the photon. Thus, the hand of a blue photon rotates faster than the hand of a red photon. As the photon travels from one position in space to another, the clock encodes its time of flight t modulo the period $1/f$. We therefore view the photon as a messenger carrying as message the position of the clock's hand. We encode the message as a two-dimensional unit vector $\mathbf{e} = (e_0, e_1) = (\cos\omega t, \sin\omega t)$. This particle model for the photon was previously used by Feynman in his theory of quantum electrodynamics.[43] Feynman used the position of the clock's hand to calculate the probability amplitudes. Although quantum electrodynamics resolves the wave-particle duality by saying that light is made of particles (as Newton originally thought), it is only able to calculate the probability that a photon will hit a detector, without offering a mechanism of how this actually happens.[43]

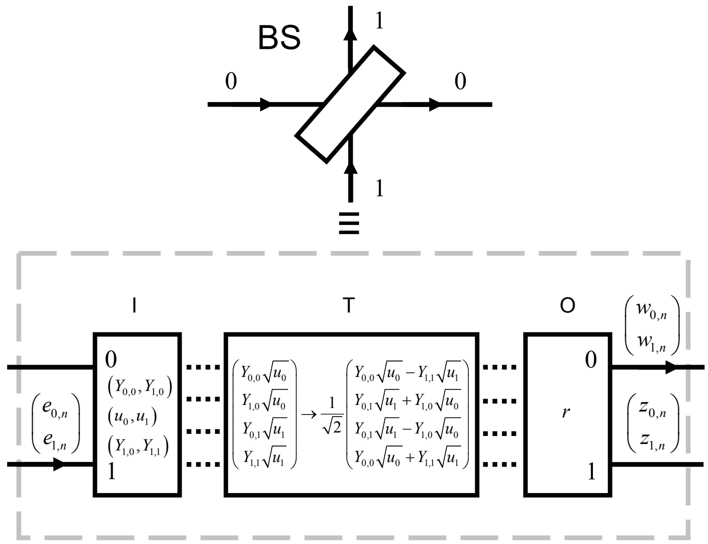

Figure 3. Diagram of a processing unit that performs an ecent-based simulation of a beam splitter (BS). The solid lines represent the input and out channels of the BS. The presence of a message is indicated by arrows on the corresponding channel line. The dashed lines indicate the data flow within the BS.

- Source: The source creates a messenger (photon), carrying a message as described above, and waits until its message has been processed by a detector before creating the next messenger. Hence, there can be no direct communication between the messengers. Therefore, the simulation model (trivially) satisfies Einsteins criterion of local causality. When a messenger is created, its internal clock time is set to zero. We label the messengers and their messages by a subscript $n \geq 0$.

- Beam splitter: The processor modeling a beam splitter consists of three stages: An input stage (I), a transformation stage (T) and an output stage (O), see Fig. 3. The input stage has two input channels labeled by $k = 0, 1$, two registers $\mathbf{Y}_k = (Y_{0,k}, Y_{1,k})$ and an internal two-dimensional vector $\mathbf{u} = (u_0, u_1)$ with the additional constraints that $u_i \geq 0$ for $i = 0, 1$ and that $u_0 + u_1 = 1$. The $(n+1)$-th messenger carrying the message $\mathbf{e}_{n+1} = (e_{0,n+1}, e_{1,n+1})$ arrives at input channel 0 or input channel 1. If the messenger arrives on input channel 0 (1), then register $\mathbf{Y}_0$ ($\mathbf{Y}_1$) stores the message brought by the messenger, that is $\mathbf{Y}_0 = (e_{0,n+1}, e_{1,n+1})$ ($\mathbf{Y}_1 = (e_{0,n+1}, e_{1,n+1})$). Note that only one of the two registers is updated when a messenger arrives at the processor. After arrival of the $(n+1)$-th messenger on input channel $k = 0, 1$ the input stage also updates its internal vector according to the rule $u_{i,n+1} = \alpha u_{i,n} + (1-\alpha)\delta_{i,k}$ where $0 < \alpha < 1$ is a parameter. The $u_{k,n}$ can be interpreted as (an estimate of) the frequency for the arrival of a messenger on input channel k and α can be interpreted as a parameter controling the learning process of this (estimate of the) frequency.[16,18]

  The transformation stage takes the six values stored in the two registers $\mathbf{Y}_0$, $\mathbf{Y}_1$ and the internal vector $\mathbf{u}$ and transforms this data according to the rule

$$\frac{1}{\sqrt{2}}\begin{pmatrix} Y_{0,0}\sqrt{u_0} - Y_{1,1}\sqrt{u_1} \\ Y_{0,1}\sqrt{u_1} + Y_{1,0}\sqrt{u_0} \\ Y_{0,1}\sqrt{u_1} - Y_{1,0}\sqrt{u_0} \\ Y_{0,0}\sqrt{u_0} + Y_{1,1}\sqrt{u_1} \end{pmatrix} \longleftarrow \begin{pmatrix} Y_{0,0}\sqrt{u_0} \\ Y_{1,0}\sqrt{u_0} \\ Y_{0,1}\sqrt{u_1} \\ Y_{1,1}\sqrt{u_1} \end{pmatrix}, \tag{13}$$

  where we have omitted the messenger label $(n+1)$ to simplify the notation. Using two complex numbers instead of four real numbers Eq. (13) can also be written as

$$\frac{1}{\sqrt{2}}\begin{pmatrix} Y_{0,0}\sqrt{u_0} - Y_{1,1}\sqrt{u_1} + i(Y_{0,1}\sqrt{u_1} + Y_{1,0}\sqrt{u_0}) \\ Y_{0,1}\sqrt{u_1} - Y_{1,0}\sqrt{u_0} + i(Y_{0,0}\sqrt{u_0} + Y_{1,1}\sqrt{u_1}) \end{pmatrix}$$

$$\longleftarrow \begin{pmatrix} Y_{0,0}\sqrt{u_0} + iY_{1,0}\sqrt{u_0} \\ Y_{0,1}\sqrt{u_1} + iY_{1,1}\sqrt{u_1} \end{pmatrix}. \tag{14}$$

Identifying a_0 with $Y_{0,0}\sqrt{u_0} + iY_{1,0}\sqrt{u_0}$ and a_1 with $Y_{0,1}\sqrt{u_1} + iY_{1,1}\sqrt{u_1}$ it is clear that the transformation Eq. (14) plays the role of the matrix-vector multiplication

$$\begin{pmatrix} b_0 \\ b_1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} a_0 + ia_1 \\ a_1 + ia_0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}, \tag{15}$$

where the (a_0, a_1) ((b_0, b_1)) denote the amplitudes of the photons in the input (output) output modes 0 and 1 of a beam splitter.[16,18]

The output stage of the processor uses the content of the four-dimensional vector in Eq. (13) to update the message carried by the $(n+1)$th messenger and directs this messenger to one of its two output channels labeled by $k = 0, 1$. The output stage sends the $(n+1)$-th messenger with message $\mathbf{w}_{n+1} = (Y_{0,0}\sqrt{u_0} - Y_{1,1}\sqrt{u_1}, Y_{0,1}\sqrt{u_1} + Y_{1,0}\sqrt{u_0})/\sqrt{2}$ through output channel 0 if $w_{0,n+1}^2 + w_{1,n+1}^2 > r$ where $0 < r < 1$ is a uniform random number. Otherwise, it sends the message $\mathbf{z}_{n+1} = (Y_{0,1}\sqrt{u_1} - Y_{1,0}\sqrt{u_0}, Y_{0,0}\sqrt{u_0} + Y_{1,1}\sqrt{u_1})/\sqrt{2}$ through output channel 1.

- Detector: In the MZI experiment the detectors are counters that simply count the number of messengers (photons) that they receive. In the modified MZI experiment we propose here, the detectors D_0 and D_1 each have two counters, one for counting detection events corresponding to the parameter setting $x = -1$ and one for counting detection events corresponding to the parameter setting $x = +1$. Hence, in total we have four counters: $N_0(x = -1)$, $N_0(x = +1)$, $N_1(x = -1)$ and $N_1(x = +1)$. Recall that x is a parameter of which the value is always known with certainty.

5. SIMULATION RESULTS

First we consider the case in which we do not group the detection events according to the value of x. In other words, independent of the procedure to choose x, we discard the information about x. In Fig. 4, we present results for the normalized frequency F_0' as a function of $\phi_0 \in [0, 2\pi]$ for the experiment in which x is changed according to the systematic procedure with $K = 1, 10, N$, where the number of particles $N = 10^6$. The detection events are not grouped (associated) with the value of x at the time of the detection event. From these data, we see that

1. For $K = 1$ (solid triangles), that is when x alternates for each photon entering the MZI, the event-based corpuscular model reproduces the statistical results of quantum theory (solid line connecting the triangles, as given by Eq. (8)).
2. For $K = 10$ (open triangles), that is when x alternates for each ten photons entering the MZI, there is excellent agreement between the simulation data and the results of quantum theory (solid line connecting the triangles, as given by Eq. (8)).
3. For $K = N$ (bullets), that is for fixed $x = +1$, $F_0' = F_0''(x = +1)$ and the event-based corpuscular model reproduces the statistical results of quantum theory (dotted line connecting the bullets, as given by Eq. (9)).

Summarizing: For fixed x, the results of the event-based corpuscular model are in excellent agreement with Eqs. (9) and (10), that is with quantum theory. Varying x and without grouping the detection events according to the value of x, the frequencies at detector D_0 obtained from the event-based corpuscular model agree perfectly with the probability distribution Eq. (8) predicted by quantum theory. The results do not depend on the number of photons (K) per change of x.

Next we consider the case in which we group the detection events according to the value of x. Unlike quantum theory, which predicts the probability distributions to be independent of details of the sequence of x-values if the detection events are grouped according to the value of x (see Eqs. (6) and (7)), the event-based corpuscular model

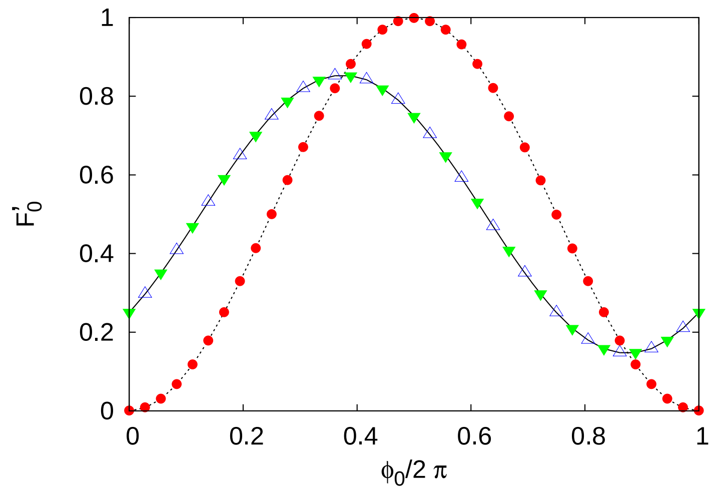

Figure 4. Results for the normalized frequency F_0' of detection events that are not grouped according to the value of x. Data are obtained from simulations employing fully classical, locally causal, corpuscular models[16,18] for all the components of the MZI experiment shown in Fig. 2. For each value of ϕ_0, $N = 10^6$ input events were generated and the model parameter $\alpha = 0.99$. Dotted line: Prediction of quantum theory, see Eq. (9); Solid line: Prediction of quantum theory, see Eq. (8); Bullets: Simulation data for $x = +1$ fixed. Solid triangles: Simulation data for the case that x changes sign ($x = +1, -1, +1, \ldots$) with each photon emitted, corresponding to the systematic procedure for changing x with $K = 1$. Open triangles: Simulation data for the case that x changes sign with every ten photons emitted, corresponding to the systematic procedure for changing x with $K = 10$.

of a MZI makes specific predictions for the frequencies observed at detector D_0 that depend on the procedure to change x and on the number of particles K that pass through the MZI while x is constant.

By construction,[16,18] the event-based corpuscular model produces detection events with frequency $I_0(x)$ if the particle travels along the upper arm of the MZI. However, if the particle takes the lower arm and x changes before the particle is detected, the detection event will be associated with the "wrong" value of x. From the description of the event-based corpuscular model, it follows directly that the observed frequencies at detector D_0 are given by

$$\widetilde{I}_0(x=+1) = \frac{1-E}{2}\sin^2\frac{\phi_0}{2} + \frac{E}{2}\sin^2\frac{\phi_0+\pi/2}{2}, \tag{16}$$

$$\widetilde{I}_0(x=-1) = \frac{1-E}{2}\sin^2\frac{\phi_0+\pi/2}{2} + \frac{E}{2}\sin^2\frac{\phi_0}{2}, \tag{17}$$

where $0 \le E \le 1$ is the rate of making wrong associations.

In Fig. 5(left), we present results for the normalized frequency $F_0(x = +1)$ as a function of $\phi_0 \in [0, 2\pi]$ for the experiment in which x is changed according to the systematic procedure with $K = 1, 10$, where the number of particles $N = 10^6$. The detection events are grouped (associated) with the value of x at the time of the detection event. From these data, we conclude that

1. For $K = 1$ (solid triangles), that is when x alternates for each photon entering the MZI, the event-based corpuscular model predicts significant deviations from the results of quantum theory (dotted line, Eq. (6)). There is excellent agreement between the simulation data and Eq. (16) (solid line through the solid triangles) with $E = 0.333$.

2. For $K = 10$ (open triangles), that is when x alternates for each ten photons entering the MZI, the difference between the data generated by the event-based corpuscular model and the results of quantum theory (dotted line, Eq. (6)) becomes rather small. There is excellent agreement between the simulation data and Eq. (16) with $E = 0.100$ (solid line through the open triangles).

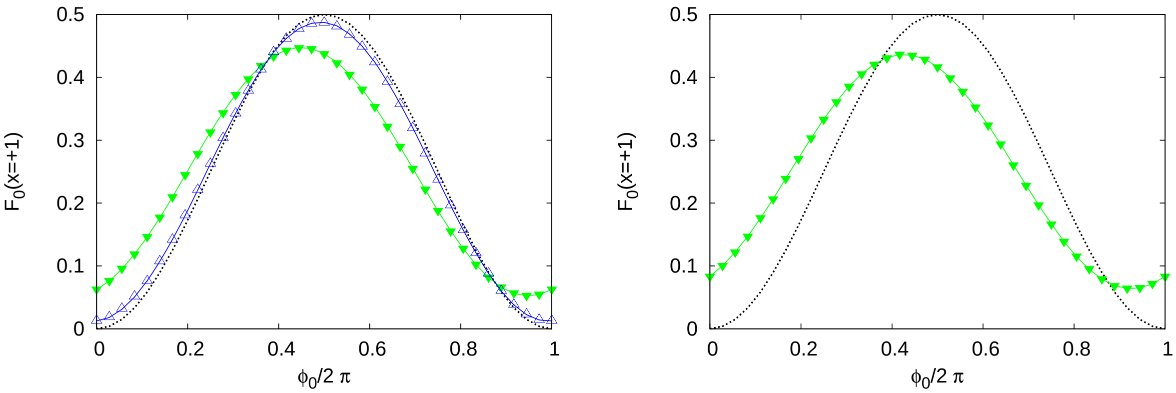

Figure 5. Left: Results for the normalized frequency $F_0(x=+1)$ of detection events that are grouped according to the value of x. Data are obtained from simulations employing fully classical, locally causal, corpuscular models[16,18] for all the components of the MZI experiment shown in Fig. 2. For each value of ϕ_0, $N = 10^6$ input events were generated and the model parameter $\alpha = 0.99$. Dotted line: Prediction of quantum theory, see Eq. (6); Solid triangles: Simulation data for the case that x changes sign $(x = +1, -1, +1, \ldots)$ with each photon emitted, corresponding to the systematic procedure for changing x with $K = 1$. The solid line through the data points is given by Eq. (16) with $E = 0.333$. Open triangles: Simulation data for the case that x changes sign with every ten photons emitted, corresponding to the systematic procedure for changing x with $K = 10$. The solid line through the data points is given by Eq. (16) with $E = 0.100$. Right: Same as left except that x is changed according to the random procedure with $K = 1$ and that the solid line through the triangles is given by Eq. (16) with $E = 1/(2+2K) = 1/4$.

Simulations (data not shown) confirm the intuitively evident expectation that as the number of photons K between changes of x increases, the data produced by the event-based corpuscular model converge to the prediction of quantum theory Eq. (6). This also follows directly from the analytic expression Eq. (16) because $E \to 0$ if $K \to N$.

In Fig. 5(right), we present simulation data for the case in which x is changed according to the random procedure with $K = 1$. Qualitatively, the results are the same as when x changes systematically (see Fig. 5)(left). However, the rate E is different. For $K = 1$, $E = 0.333$ for the systematic procedure and $E = 0.25$ for the random procedure. In the case of the random procedure, simulation data for various K (not shown) are rather accurately represented by Eq. (16) with $E = 1/(2+2K)$. Although the quantitative differences between the normalized frequencies $F_0(x=+1)$ computed for the event-based corpuscular model and quantum theory are larger if the systematic procedure for changing x is used instead of the random procedure, the data obtained with the random procedure for changing x might be more useful for comparing with the outcomes of laboratory experiments, as discussed in the next section.

Summarizing: In order to see a difference between the interference patterns predicted by quantum theory and the event-based corpuscular models, a key factor in the proposed experiment is that the detection events are associated with the value of x at the time of the detection event. If the detection events are grouped according to the value of x, the frequencies of events at detector D_0 as obtained from the event-based corpuscular model are given by Eqs. (16) and (17). Note that the difference with Eq. (8) is only in the prefactors ($E/2$ and $(1-E)/2$ with $0 \le E \le 1$ instead of $1/2$) which depend on the details of the sequence of x-values.

6. APPLICABILITY OF QUANTUM THEORY TO EXPERIMENTS WITH A CONTINUOUSLY CHANGING PREPARATION PROCEDURE

As already mentioned, quantum theory gives an accurate description of the statistics of an experiment in which the procedure of preparing the particles before they are detected does not change during the experiment. As the experiment that we propose can be performed such that this condition is not satisfied, it is of interest to perform this experiment and verify that it agrees with the quantum theoretical prediction. If the proposed experiment would show deviations from the quantum theoretical prediction, this finding does not refute quantum theory as such: It provides experimental evidence that quantum theory cannot be applied to statistical experiments in which the procedure of preparing the particles before they are detected changes in the course of the experiment.

The event-based corpuscular model[16,18] operates on a level that quantum theory has nothing to say about and it can easily cope with a preparation procedure that changes with each particle ($K = 1$). As this model reproduces the results of quantum theory under the condition that the preparation procedure is fixed (K and N large),[16,18] conventional quantum optics experiments cannot refute the event-based corpuscular model. However, as Fig. 5 shows, the proposed MZI experiment with a phase difference alternating between ϕ_0 and $\phi_0 + \pi/2$ (see Fig. 5(left)) or with a phase difference randomly taking the values ϕ_0 and $\phi_0 + \pi/2$ (see Fig. 5(right)), can discriminate between quantum theory and the event-based corpuscular model[16,18] if the detection events are associated with the value of x at the time of the detection event, at least in principle. Recall that if the detection events are not grouped according to the value of x at the time of detection, both quantum theory and the event-based corpuscular model yield the same interference pattern (see Fig. 4).

To appreciate the subtilities that are involved, it is necessary to recognize that there are other experiments in which the preparation procedure is not fixed in time and for which we do not expect the predictions of quantum theory to deviate from the experimental results, independent of the pace at which the preparation procedure changes. As an example, consider Wheeler's delayed choice experiment with single-photons.[19] In this experiment, the random choice between the open and closed configuration of the interferometer with each passage of a photon does not affect the agreement of the experimental observations with predictions of quantum theory.[19] The reason is that a passage of a photon in the open configuration has no causal effect on the passage of a photon in the closed configuration. As the event-based corpuscular approach reproduces the results of quantum theory for Wheeler's delayed choice experiment[20] this experiment[19] cannot be used to refute the event-based corpuscular model.

The experiment that we propose in this paper is fundamentally different from e.g. Wheeler's delayed choice experiment with photons in that the second beam splitter, being the physical cause for interference to occur at all, is present at all times and that, in a corpuscular picture, the physical state of a beam splitter may change with each photon passing through it.

7. REALIZATION IN A LABORATORY EXPERIMENT

We now address some issues that become relevant when the proposed experiment is performed in practice. Essential for the proposed experiment to refute the event-based corpuscular model or to show the aforementioned limitation of quantum theory is that the rate at which photons are emitted is lower than the rate at which the time-of-flight in the upper arm of the interferometer (see Fig. 2) is being switched between two different values. Assuming that there is uncertainty about whether or not the source emits a photon and assuming that the frequency of these pulses is incommensurate with the frequency with which x changes, to describe the experiment we may use the model in which x is changed according to the random procedure with $K = 1$, see Fig. 6(left). We emphasize that for the proposed experiment to be successful, the time-of-flight of a photon from the source to detector should be much less than the time between changes of x such that there is a one-to-one correspondence between the value of x and the photon (independent of whether it is actually detected). Equally essential is that the procedure to change the time-of-flight of the particles traveling in the upper arm of the MZI does not alter the particle's direction towards the second beam splitter.

Refuting the event-based corpuscular model[16,18] or to demonstrate the aforementioned limitation of quantum theory by an experiment will be a real challenge. The central issue is to collect and analyze the experimental data properly. To see this, consider the expression for the normalized frequency of events on output channel 0. In general, that is for $\phi_1(x = +1)\text{mod}2\pi = 0$ and $\phi_1(x = -1)\text{mod}2\pi = \delta$, the event-based corpuscular model predicts

$$\widetilde{I}_0(x=+1) \quad = \quad \frac{1-E}{2}\sin^2\frac{\phi_0}{2} + \frac{E}{2}\sin^2\frac{\phi_0-\delta}{2} = \frac{1-\Delta\cos(\phi_0-\psi)}{4}, \tag{18}$$

where $\psi = \arctan(E\sin\delta/(1-E+E\cos\delta))$ and $\Delta = (2E^2 - 2E + 1 + 2E(1-E)\cos\delta)^{1/2}$. From Eq. (18) it follows directly that a least-square fit of a sinusoidal function to the data produced by the event-based corpuscular

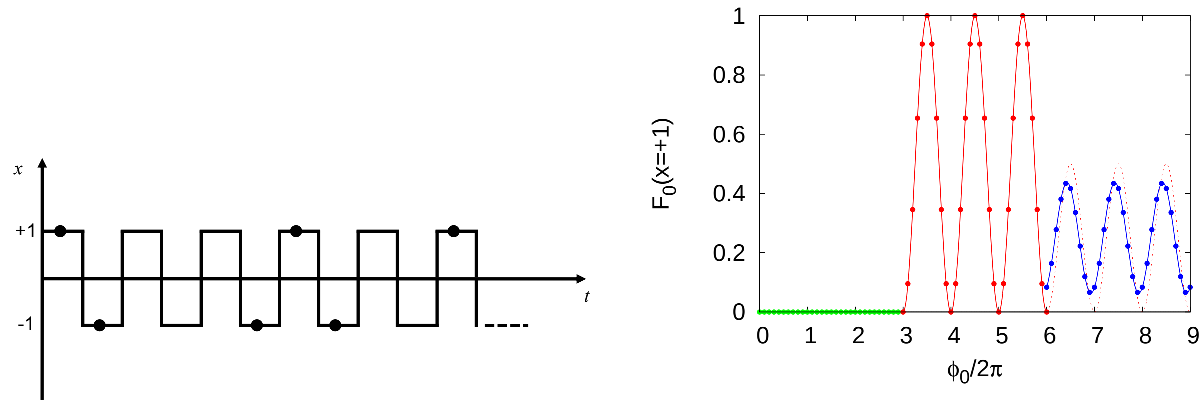

Figure 6. Left: In the realization of the proposed experiment the variable x, taking the values +1 and -1, can be changed alternately in time t at a given fixed rate. The rate at which the photons (solid circles) are emitted is assumed to be lower than the rate at which x is changed. Assuming that it is uncertain whether the source emits a photon with each trigger pulse, this experiment is similar to the case where x is changed according to the random procedure with $K = 1$. Right: Results of the normalized frequency $F_0(x = +1)$ in a three-staged MZI experiment in which the detection events are grouped according to the value of x. First stage ($0 \leq \phi_0/2\pi < 3$): $x = -1$ fixed. Second stage ($3 \leq \phi_0/2\pi < 6$): $x = +1$ fixed. Third stage ($6 \leq \phi_0/2\pi < 9$): x is changed according to the systematic procedure with $K = 1$. For each value of ϕ_0, $N = 10^6$ input events were generated and the model parameter $\alpha = 0.99$. Symbols denote the simulation results. The solid lines are given by Eq. (9) for the first two stages and by Eq. (16) with $E = 0.33$ for the third stage. The dotted line is given by quantum theory (Eq. (9) for stages one and two and Eq. (6) for stage three.

model could lead to the conclusion that, independent of the values of E and δ, this data is described by quantum theory, albeit with a reduced visibility ($|\Delta| < 1$). Thus, this naive procedure to analyse data of single-photon interference experiments cannot lead to a refutation of the event-based corpuscular model nor can it be used the test the applicability of quantum theory to event-based experiments. However, the proposed experiment may be carried out such that there is a chance that the event-based corpuscular model[16,18] can be refuted and/or these limitations of quantum theory can be demonstrated.

Specifically, for each pulse applied to the single photon source (labeled by the subscript n), the experiment should collect the triples $\{x_n, d_{0,n}, d_{1,n}\}$ for $n = 1, \ldots, N, N+1, \ldots, 2N, 2N+1, \ldots, 3N$ where $d_{k,i} = 1$ if detector D_k, $k = 0, 1$ fired (within a properly chosen time window) and $d_{k,i} = 0$ otherwise. Note that recording both $d_{0,n}$ and $d_{1,n}$ is required for ensuring the single-particle character of the experiment.[2] For each value of ϕ_0, in the first stage (the first N pulses), $x = -1$ is kept fixed while in the second stage of N pulses $x = +1$ kept fixed. Finally, to mimic a random sequence of x-values, in the third stage of N pulses x should change much faster than the pulse rate at which single photons are emitted. Assuming that the MZI is stable enough to allow a sufficient amount of triples to be collected and that the photon flux during the three stages is the same, comparison of the number of detection counts of the first and second stage with the one of the third stage, should or should not (if quantum theory applies) reveal a significant change in the detection counts (see Fig. 6(right)). In other words, performing these three stages in one experimental run should allow one to see a reduction in visibility and a shift of the sinusoidal curve in the stage in which x changes with respect to the two other stages in which x is fixed. In experiment this staged procedure may be necessary to compare the reduced visibility (due to experimental limitations) for the cases with fixed and varying x (which according to quantum theory should all be the same). Changing the order of the stages and repeating the experiment should provide some information about the reproducibility of the experimental data.

It will not have escaped the reader that we have not made any assumption about the efficiency of detecting the photons. Although for photons this efficiency may be quite low,[44] this should not affect the conclusions that can be drawn from the experimental data as long as this data is not contaminated by a significant fraction of dark counts. The dark counts may be reduced by using a source emitting pairs of photons in different directions and by correlating the detection times of the photons detected on detector D_0 or D_1 placed behind the MZI with those detected on the detector D placed on the other side of the source (see Fig. 2).

Although our proposal has been formulated in terms of single-photon experiments, it should be evident that, at least in theory, one can replace "photon" by "neutron" without altering the conclusions. In fact, a neutron experiment which resembles the modified MZI experiment we propose here has been performed,[45] but the switching of the conditions was not correlated with the detection events.

We hope that our proposal will stimulate experimenters to take up the challenge to determine the extent to which quantum theory provides a description of event-based processes that goes beyond statistical averages or to refute event-based corpuscular models that, without invoking any concept of quantum theory, reproduce the statistical results of quantum theory.

8. CONCLUSIONS

We have proposed a modified single-photon Mach-Zehnder interferometer experiment in which the preparation procedure of the photons in the Mach-Zehnder interferometer (before detection) is changing in time. Given

(i) the general belief that quantum theory can be used to describe all single-photon experiments,

(ii) the fact that quantum theory gives an accurate description of the statistics of an experiment in which the procedure of preparing the particles before they are detected does not change during the experiment,

(iii) the fact that the frequency distributions produced by the event-based corpuscular models cannot be distinguished from those predicted by quantum theory for the single-photon experiments performed so far,

(iv) the fact that the interference patterns of the event-based corpuscular model for the proposed experiment do not agree with those predicted by quantum theory,

makes this an interesting experiment to be carried out. The key question is: Which interference patterns are produced by a real laboratory experiment?

9. ACKNOWLEDGEMENT

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