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Corpuscular event-by-event simulation of quantum optics experiments: application to a quantum-controlled delayed-choice experiment

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Abstract
A corpuscular simulation model of optical phenomena that does not require knowledge of the solution of a wave equation of the whole system and reproduces the results of Maxwell's theory by generating detection events one by one is discussed. The event-based corpuscular model gives a unified description of multiple-beam fringes of a plane parallel plate and a single-photon Mach–Zehnder interferometer, Wheeler's delayed choice, photon tunneling, quantum eraser, two-beam interference, Einstein–Podolsky–Rosen–Bohm and Hanbury Brown–Twiss experiments. The approach is illustrated by applying it to a recent proposal for a quantum-controlled delayed choice experiment, demonstrating that also this thought experiment can be understood in terms of particle processes only.

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(Some figures may appear in color only in the online journal)

1. Introduction

Quantum theory has proven extraordinarily powerful in describing the statistical properties of a large number of laboratory experiments. Conceptually, it is straightforward to use the quantum theoretical formalism to calculate numbers that can be compared with experimental data, at least if these numbers refer to statistical averages. However, a fundamental problem appears if an experiment provides access to the individual events that collectively build the statistical average. Prime examples are the single-electron two-slit experiment [1], neutron interferometry experiments [2] and similar experiments in optics where the click of the detector is identified with the arrival of a single photon [3]. Although quantum theory provides a recipe to compute the frequencies for observing events, it does not account for the observation of the individual detection events themselves [4, 5]. For a recent review of various approaches to the quantum measurement problem and an explanation of it within the statistical interpretation, see [6].

From the viewpoint of quantum theory, the central issue is how it can be that experiments yield definite answers. As stated by Leggett [7]: 'In the final analysis, physics cannot forever refuse to give an account of how it is that we obtain definite results whenever we do a particular measurement'.

This paper is not about interpretations or extensions of quantum theory. It provides a brief account of a very different approach for dealing with the fact that experiments yield definite results. The latter, which is intimately linked to human perception, is considered as fundamental. We call these definite results 'events'. Instead of trying to fit the existence of these events into some formal, mathematical theory, we change the paradigm by directly searching for the rules that transform events into other events and, by repeated application, yield frequency distributions of events that agree with those predicted by quantum theory. Obviously, such rules cannot be derived from quantum theory or, as a matter of fact, any theory that is probabilistic in nature simply because these theories do not entail a procedure (= algorithm) to produce events themselves.

The event-based approach has successfully been used to perform discrete-event simulations of the single beam splitter and Mach–Zehnder interferometer experiment of Grangier *et al* [8] (see [9–11]), Wheeler's delayed choice experiment of Jacques *et al* [12] (see [11, 13, 14]), the quantum eraser experiment of Schwindt *et al* [15] (see [11, 16]), double-slit and two-beam single-photon interference experiments and the single-photon interference experiment with a Fresnel biprism of Jacques *et al* [17] (see [11, 18]), quantum cryptography protocols (see [19]), the Hanbury Brown–Twiss experiment of Agafonov *et al* [20] (see [11, 21]), universal quantum computation (see [22, 23]), Einstein–Podolsky–Rosen–Bohm-type experiments of Aspect *et al* [24, 25] and Weihs *et al* [26] (see [11, 27–32]) and the propagation of electromagnetic plane waves through homogeneous thin films and stratified media (see [11, 33]). An extensive review of the simulation method and its applications is given in [11].

A detailed discussion of the discrete-event approach cannot be fitted into this short paper. Therefore, we have chosen to illustrate the approach by applying it to a recent proposal for a quantum-controlled Wheeler delayed choice experiment [34]. We demonstrate that also this thought experiment can be understood in terms of event-based, particle-like processes only. The presentation is sufficiently detailed such that the reader who is interested can reproduce our results.

2. Wheeler's delayed-choice experiment

Particle–wave duality, a concept of quantum theory, attributes to photons the properties of both wave and particle behaviors depending on the circumstances of the experiment [4]. The particle behavior of photons has been shown in an experiment composed of a single beam splitter (BS) and a source emitting single photons and pairs of photons [8]. The wave character has been demonstrated in a single-photon Mach–Zehnder interferometer (MZI) experiment [8]. The layout of such an experiment is shown in figure 1. By adding a device which controls the presence or absence of the second beam splitter BS2, this setup can be used to perform a delayed-choice experiment. Originally, Wheeler proposed a double-slit gedanken experiment in which the decision to observe wave or particle behavior is made after the photon has passed the slits [35]. Similarly, in the MZI experiment, the decision to remove and place BS2 at the intersection of paths 0 and 1 can, in principle, be made after the photon has passed BS1. The conclusion is that the pictorial description

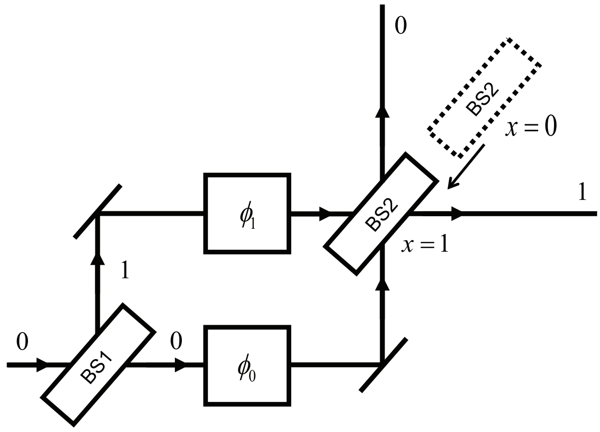

Figure 1. Diagram of a standard Wheeler delayed-choice experiment with a Mach–Zehnder interferometer. Photons enter the interferometer via 50–50 beam splitter 1 (BS1). In the wave picture, the partial wave traveling along path 0 (1) acquires a phase shift ϕ_0 (ϕ_1). The variable $x = 0, 1$ controls the presence of 50–50 beam splitter 2 (BS2). If BS2 is not in place ($x = 0$, indicated by the dashed rectangle) the partial waves do not interfere and the probability to observe the photon in path 0 or 1 does not depend on the phase shifts. If BS2 is in place ($x = 1$, indicated by a solid rectangle) the partial waves interfere and the probability to observe the photon in path 0 or 1 is given by $(1+\cos(\phi_0-\phi_1))/2$ or $(1-\cos(\phi_0-\phi_1))/2$, respectively.

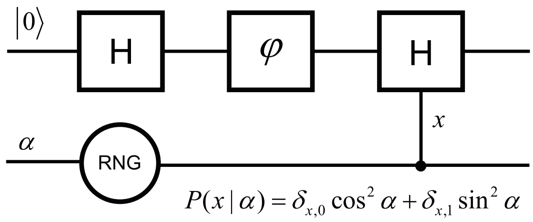

Figure 2. Quantum gate representation of the standard Wheeler delayed-choice experiment with a Mach–Zehnder interferometer (see figure 1). The first Hadamard gate **H** acts as a 50–50 beam splitter, changing the state $|0\rangle$ into the state $(|0\rangle+|1\rangle)/\sqrt{2}$. The phase gate φ changes the amplitude of the state $|1\rangle$ by $e^{i\varphi}$. The second (controlled) Hadamard gate **H** act as a 50–50 beam splitter if the control variable $x = 1$ or passes the photons unaltered if $x = 0$. The angle α determines the probability that the control variable x is 1. A pair of detectors (not shown) signals the presence of a photon in the state $|0\rangle$ or $|1\rangle$ and with each detected photon the value of x is being recorded.

of this experiment defies common sense: the behavior of the photon in the past is said to be changing from a particle to a wave or vice versa.

3. Quantum-controlled delayed-choice experiment

It is of interest to enquire what happens if the variable x that controls the presence of BS2 (see figure 1) or, equivalently, the controlled Hadamard gate (see figure 2) is replaced by a quantum two-state system [34]. In a sense, one could then view the experiment as a simple example of a quantum-controlled experiment [34]. The original proposal of the quantum-controlled delayed-choice experiment [34] is formulated in a notation that is commonly used in the quantum computer literature [36]. To facilitate a comparison with this work, we also adopt this notation from now on.

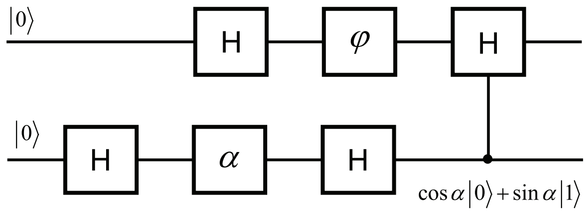

Figure 3. Quantum gate representation of the quantum version of the Wheeler delayed-choice experiment with a Mach–Zehnder interferometer [34]. Reading from left to right, the first Hadamard gate **H** on the top line acts as a 50–50 beam splitter and the phase gate φ changes the amplitude of the state $|1\rangle$ by $\mathrm{e}^{\mathrm{i}\varphi}$. The second (controlled) Hadamard gate **H** on the top line acts as a 50–50 beam splitter if the state of the ancilla is $|1\rangle$ or passes the photons unaltered if that state is $|0\rangle$. Initially in the state $|0\rangle$, the ancilla is prepared in a uniform superposition of the states $|0\rangle$ and $|1\rangle$ by another interferometer circuit (bottom line) in which the phase gate α changes the amplitude of the ancilla state $|1\rangle$ by $\mathrm{e}^{\mathrm{i}\alpha}$. The angle α determines the probabilities of the states $|0\rangle$ and $|1\rangle$. A pair of detectors (not shown) signals the presence of the photon in the state $|0\rangle$ or $|1\rangle$. Similarly, another pair of detectors (not shown) signals the presence of the ancilla in the state $|0\rangle$ or $|1\rangle$.

First, in figure 2 we show the quantum gate diagram that is equivalent to the standard delayed-choice experiment depicted in figure 1. The main change, irrelevant from a conceptual point of view, is to replace the beam splitters by Hadamard gates. In [34], it is proposed to replace the classical random variable x in figure 2 by a qubit, conventionally called ancilla, that can be in a superposition of the states $|0\rangle$ and $|1\rangle$. As shown in figure 3, the state of the ancilla controls the operation of the last Hadamard gate on the top line. In our implementation, we have chosen to include a preparation procedure for the state of the ancilla, as indicated in figure 3.

For completeness and comparison with the event-by-event simulation data, we give the quantum-theoretical description of this experiment in terms of the state $|vu\rangle = |v\rangle \otimes |u\rangle$ where $u, v = 0, 1$ label the basis states and $|u\rangle$ and $|v\rangle$ denote the state of the ancilla and photon, respectively. The amplitudes at the input $\mathbf{a} = (a_{00}, a_{01}, a_{10}, a_{11})^{\mathrm{T}}$ and the output $\mathbf{b} = (b_{00}, b_{01}, b_{10}, b_{11})^{\mathrm{T}}$ of the experiment depicted in figure 3 are related by

$$\mathbf{b} = \begin{pmatrix} 1 & 0 & a & 0 \\ 0 & a & 0 & a \\ 1 & 0 & 1 & 0 \\ 0 & -a & 0 & a \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \mathrm{e}^{\mathrm{i}\varphi} \end{pmatrix} \times \begin{pmatrix} a & 0 & a & 0 \\ 0 & a & 0 & a \\ -a & 0 & a & 0 \\ 0 & -a & 0 & a \end{pmatrix} \begin{pmatrix} a & a & 0 & 0 \\ -a & a & 0 & 0 \\ 0 & 0 & a & a \\ 0 & 0 & -a & a \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\alpha} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & a & 0 & 0 \\ -a & a & 0 & 0 \\ 0 & 0 & a & a \\ 0 & 0 & -a & a \end{pmatrix} \mathbf{a}, \qquad (1)$$

where $a = 1/\sqrt{2}$.

Reading from right to left, the matrices in equation (1) represent the action of a Hadamard operation on the ancilla, a phase shift (by α) operation on the ancilla, another Hadamard operation on the ancilla, a Hadamard operation on the photon, a phase shift (by φ) operation on the photon and a controlled (by the ancilla) Hadamard operation on the photon. Note that all these operations only affect the state, i.e. the wave function, which describes the statistical properties of the whole system and cannot be interpreted as having causal effects on a particular particle without running into conceptual and logical problems [4].

For the case at hand, $a_{00} = 1$ and all other a's are zero. Then it follows from equation (1) that the probability to detect a pair (photon, ancilla) in the state $|vu\rangle$ is given by $p(v, u) = |b_{v,u}|^2$. More explicitly, we have [34]

$$\begin{aligned} p(v=0, u=0) &= \tfrac{1}{2}\cos^2\alpha, \\ p(v=1, u=0) &= \tfrac{1}{2}\cos^2\alpha, \\ p(v=0, u=1) &= \sin^2\alpha\cos^2\frac{\varphi}{2}, \\ p(v=1, u=1) &= \sin^2\alpha\sin^2\frac{\varphi}{2}. \end{aligned} \qquad (2)$$

Note that equation (2) is identical to the corresponding result for the standard delayed-choice experiment.

4. The simulation model

The model presented in this paper builds on earlier work [9–11, 22, 23, 27–30, 32] in which we have demonstrated that it may be possible to simulate quantum phenomena on the level of individual events without invoking concepts of quantum theory.

In our simulation approach, a messenger (representing the photon or the ancilla) carries a message (representing the phase) and is routed through the network and the various units that process the messages.

We now explicitly describe our simulation model, that is, we specify the message carried by the messengers, the algorithms that simulate the processing units and the data analysis procedure.

4.1. The messenger

Particles carry a message represented by a two-dimensional unit vector $\mathbf{y}_{k,n} = (\cos\psi_{k,n}, \sin\psi_{k,n})$, where $\psi_{k,n}$ refers to the phase of the photon. The subscript $n \geqslant 0$ labels the consecutive messages and $k = 0, 1$ labels the port of the beam splitter at which the message arrives. Every time a messenger is created, the message is initialized to $\mathbf{y}_{k,n} = (1, 0)$.

4.2. The Hadamard gate

The key element of the event-by-event approach is a processing unit that is adaptive, that is, it can learn from the messengers that arrive at its input ports [9–11]. The processing unit consists of an input stage called a deterministic learning machine (DLM) [9, 10], a transformation stage and an output stage. In experiments with single particles, the input stage receives a message on either input port $k = 0$ or $k = 1$, but never on both ports simultaneously. The arrival of a message on port 0 (1) corresponds to an event of type 0 (1). The input events are represented by the vectors $\mathbf{e}_n = (1, 0)$ or $\mathbf{e}_n = (0, 1)$ if the nth event occurred on port 0 or 1, respectively. The DLM has two sets of internal registers $(C_{k,n}, S_{k,n})$ and one internal vector $\mathbf{x}_n = (x_{0,n}, x_{1,n})$, where $x_{0,n} + x_{1,n} = 1$ and $x_{i,n} > 0$. These three two-dimensional vectors are labeled by the message number n because their content is updated every time the DLM receives a message. Thus, the DLM can only store six numbers, not more. Before the simulation starts we set $\mathbf{x}_0 = (x_{0,0}, x_{1,0}) = (\mathscr{R}, 1-\mathscr{R})$, where $\mathscr{R}$ is a uniform pseudo-random number. In a similar way, we use pseudo-random numbers to set $(C_{k,0}, S_{k,0})$ for $k = 0, 1$. Upon receiving the $(n+1)$th input event, the DLM performs the following steps: (i) it stores the message $\mathbf{y}_{k,n+1} = (\cos\psi_{k,n+1}, \sin\psi_{k,n+1})$ in its internal register $(C_{k,n+1}, S_{k,n+1})$ and (ii) it updates its internal vector according to the rule

$$x_{i,n+1} = \gamma x_{i,n} + (1-\gamma)\delta_{i,k}, \tag{3}$$

where $0 < \gamma < 1$ is a parameter that controls the learning process. By construction, $x_{0,n+1} + x_{1,n+1} = 1$ and $x_{i,n+1} \geqslant 0$.

The parameter γ affects the time the machine needs for adapting to a new situation, that is, when the ratio of particles on paths 0 and 1 changes. By reducing γ, the time to adapt decreases but the accuracy with which the machine reproduces the ratio also decreases. In the limit $\gamma = 0$, the machine learns nothing: it simply echoes the last message that it received [9, 10]. If $\gamma \to 1^-$, the machine learns slowly and accurately reproduces the ratio of particles that enter via paths 0 and 1. It is in this case that the machine can be used to reproduce, event by event, the interference patterns that are characteristic of quantum phenomena [9–11].

The transformation stage implements the specific functionality of the unit, the Hadamard operation for the case at hand. It takes as input the data stored in the two internal registers $(C_{k,n+1}, S_{k,n+1})$ $(k = 0, 1)$ and in the internal vector $\mathbf{x}_{n+1} = (x_{0,n+1}, x_{1,n+1})$ and constructs the four-dimensional vector

$$\mathbf{V} = \frac{1}{\sqrt{2}} \begin{pmatrix} C_{1,n+1}\sqrt{x_{1,n+1}} + C_{0,n+1}\sqrt{x_{0,n+1}} \\ S_{1,n+1}\sqrt{x_{1,n+1}} + S_{0,n+1}\sqrt{x_{0,n+1}} \\ C_{1,n+1}\sqrt{x_{1,n+1}} - C_{0,n+1}\sqrt{x_{0,n+1}} \\ S_{1,n+1}\sqrt{x_{1,n+1}} - S_{0,n+1}\sqrt{x_{0,n+1}} \end{pmatrix}. \tag{4}$$

Rewriting this vector as a two-dimensional vector with complex-valued entries, it is easy to show that $\mathbf{V}$ corresponds to the matrix–vector multiplication in the quantum theoretical description of the Hadamard gate [23].

The vector $\mathbf{V}$ is then passed on to the output stage which determines the output port through which the messenger leaves the unit. The output stage sends the message

$$\mathbf{w}_{0,n+1} = (\mathbf{V}_0 + \mathbf{V}_1)/(\mathbf{V}_0^2 + \mathbf{V}_1^2)^{1/2}, \tag{5}$$

through output port 0 if $\mathbf{w}_{0,n+1}^2 < \mathscr{R}$ where $0 < \mathscr{R} < 1$ is a uniform pseudo-random number. Otherwise, the output stage sends the message

$$\mathbf{w}_{1,n+1} = (\mathbf{V}_2 + \mathbf{V}_3)/(\mathbf{V}_2^2 + \mathbf{V}_3^2)^{1/2}, \tag{6}$$

through output port 1.

4.3. The controlled Hadamard gate

The event-based processor of this device is identical to that of the Hadamard gate itself except that the vector $\mathbf{V}$ is computed according to equation (4) if the control bit (called x) is 1 only. If the control bit is 0, $\mathbf{V}$ is given by

$$\mathbf{V} = \begin{pmatrix} C_{0,n+1}\sqrt{x_{0,n+1}} \\ S_{0,n+1}\sqrt{x_{0,n+1}} \\ C_{1,n+1}\sqrt{x_{1,n+1}} \\ S_{1,n+1}\sqrt{x_{1,n+1}} \end{pmatrix}. \tag{7}$$

4.4. The phase gate

The unit that performs the phase shift by an angle ϕ changes the message $\mathbf{y}_{k,n}$ according to the rule

$$\mathbf{y}_{0,n} \leftarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\mathbf{y}_{1,n} \leftarrow \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \mathbf{y}_{1,n}. \tag{8}$$

As a result the message is rotated by ϕ if the particle traveled via path 1.

4.5. The simulation procedure

For each pair (α, φ), $N = 10\,000$ pairs of messengers (one for the photon, and one for the ancilla) are sent through the network (see figure 3) of processing units. A messenger that appears on an output line of the network exits either via port 0 or via port 1 and never via both ports simultaneously. With each pair of messengers that emerges from the network, the corresponding counter is incremented, that is, no events are being discarded. In other words, we assume that the efficiency of the detectors is 100%. After all pairs have been processed, dividing the value of one of the counters by N yields the normalized frequency for observing a pair (photon, ancilla) in the corresponding output ports.

5. Simulation results

In figure 4, we show the results of the event-based simulation of the quantum-controlled delayed-choice experiment for a fixed value $(\alpha = \pi/3)$ of the parameter that determines the probability $(\sin^2\alpha)$ that the ancilla is in the state $|1\rangle$. As the solid lines in figure 4 are predictions of quantum theory, see equation (2), it is clear that the event-based simulation reproduces the results of quantum theory for this particular value of α.

In figure 5, we plot the difference between the event-based simulation results and the prediction of quantum theory, given by equation (2). The differences are at the 1% level, as it should be on the basis of standard statistical arguments. Therefore, we may conclude that the event-by-event approach reproduces the statistical distributions of quantum theory for the quantum-controlled delayed choice experiment.

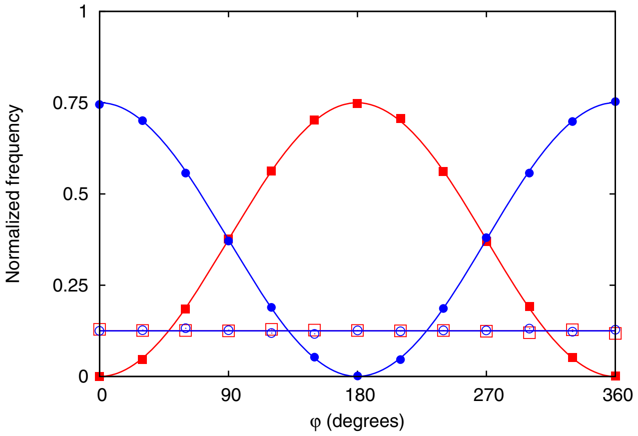

Figure 4. The normalized frequency of observing a photon in path 0 (squares) or 1 (circles) conditioned on the observation of the ancilla in path 0 (open symbols) or 1 (closed symbols) for the case in which $\alpha = \pi/3$. The solid lines are the prediction of quantum theory, see equation (2). The number of emitted and detected events per φ is 10 000. The DLM control parameter $\gamma = 0.99$.

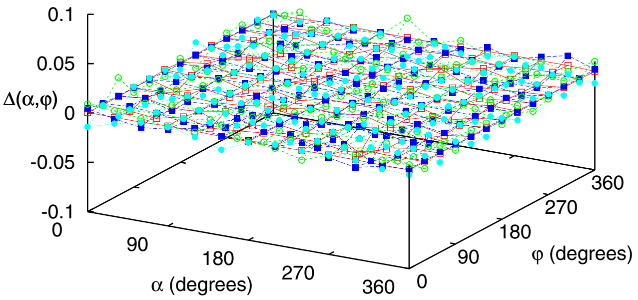

Figure 5. The difference $\Delta(\alpha, \varphi)$ between the quantum theoretical result equation (2) and the data obtained from an event-by-event simulation of the quantum circuit shown in figure 3. The number of emitted and detected events per pair (α, φ) is 10 000. The DLM control parameter $\gamma = 0.99$. The differences fluctuate at the 1% level. Open squares: photon detected in path 0, ancilla detected in path 0; closed squares: photon detected in path 1, ancilla detected in path 0; open circles: photon detected in path 0, ancilla detected in path 1; closed circles: photon detected in path 1, ancilla detected in path 1. Lines connecting markers are a guide to the eye.

6. Discussion

Instead of discussing our event-by-event simulation approach for optical phenomena in full generality, in this paper we have opted to explain in detail how the approach is applied to a specific example, a quantum-controlled delayed-choice experiment [34]. We hope that this helps us to understand the key feature of our approach, namely that it builds, one by one, the statistical distributions of quantum theory without knowing about the latter.

The successful simulation of the quantum-controlled delayed-choice experiment [34] adds to the many examples for which the event-by-event simulation method yields the correct statistical distributions. Of course, the event-based approach, being free from concepts such as particle–wave duality, does not suffer from the conflicts with everyday logic that arise in the quantum-theoretical description of the delayed-choice experiment. In particular, there is no need to invoke the thought that in this experiment, the character of the photon needs to be changed in the past.

Finally, it should be noted that although the discrete-event algorithm can be given an interpretation as a realistic cause-and-effect description that is free of logical difficulties and reproduces the statistical results of quantum theory, at present the lack of relevant data makes it impossible to decide whether or not such algorithms are realized by nature. Only new, dedicated experiments that provide information beyond the statistics can teach us more about this intriguing question.

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