Eur. Phys. J. B **53**, 139–142 (2006)
DOI: 10.1140/epjb/e2006-00364-9

THE EUROPEAN PHYSICAL JOURNAL B

A local realist model for correlations of the singlet state

K. De Raedt[1], K. Keimpema[2], H. De Raedt[2,a], K. Michielsen[2], and S. Miyashita[3]

[1] Department of Computer Science, University of Groningen, Blauwborgje 3, 9747 AC Groningen, The Netherlands
[2] Department of Applied Physics, Materials Science Centre, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands
[3] Department of Physics, Graduate School of Science, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan

Received 6 July 2006
Published online 6 October 2006 –

Abstract. Can quantum correlations of the singlet state be produced by two separate subsystems which have interacted in the past but do not communicate? We show that, using a locally causal realist model of the Einstein-Podolsky-Rosen-Bohm experiment, the answer is affirmative if coincidence in time is used to decide which detection events are stemming from a single two-particle system, the criterion employed in all experimental realizations of the Einstein-Podolsky-Rosen-Bohm gedanken experiment.

PACS. 03.65.-w Quantum Mechanics – 02.70.-c Computational Techniques – 03.65.Ud Entanglement and quantum nonlocality – 03.65.Ta Foundations of quantum mechanics

Rapid Note

Many experimental realizations and quantum mechanical descriptions of the Einstein-Podolsky-Rosen gedanken experiment [1] adopt the model proposed by Bohm [2]. In this model, sketched in Figure 1, a source emits pairs of particles with opposite magnetic moments. The two particles separate spatially and propagate in free space to an observation station in which they are detected. As the particle arrives at station $i = 1, 2$, it passes through a Stern-Gerlach magnet [3]. The Stern-Gerlach magnet deflects the particle, depending on the orientation of the magnet and the magnetic moment of the particle. The deflection defines the spin $\mathbf{S} = \pm 1/2$ of the particle [3]. As the particle leaves the Stern-Gerlach magnet, it generates a signal in one of the two detectors. The firing of a detector corresponds to a detection event.

The fundamental problem, first posed by EPR [1] in a different form, is to explain how individual events, registered by different detectors in such a way that a measurement on one particle does not have a causal effect on the result of the measurement on another particle (Einstein's criterion on local causality), exhibit two-particle quantum correlations that are found in experiments [4–11]. In this paper, we present a solution of this fundamental problem by constructing a computer simulation model of the Einstein-Podolsky-Rosen-Bohm (EPRB) experiment inspired by the experimental realization with photons performed by Weihs et al. [9], an experiment that comes very close to realizing the EPR *gedanken* experiment.

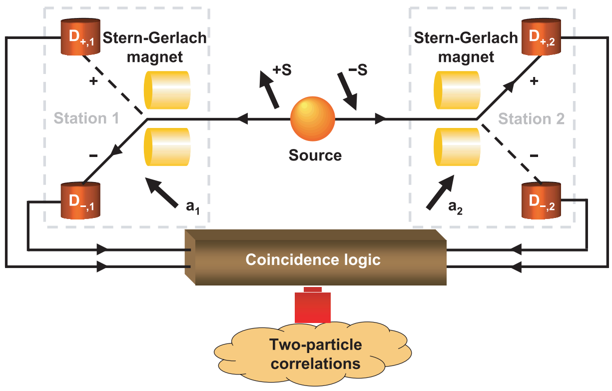

Fig. 1. (Color online) Schematic diagram of an EPRB experiment with magnetic particles [2].

Real experiments require a criterion to decide which events, registered in stations 1 and 2, correspond to the detection of particles belonging to a pair (a single two-particle system). In EPRB-experiments this criterion is the coincidence in time of the events [9,12]. Therefore, in real experiments, each station has its own clock that assigns a time-tag to each detection event. Effectively, this procedure discretizes time in intervals of a width that is determined by the time-tag resolution τ [9]. In this paper, we consider ideal experiments only. Therefore, we assume that the detectors register all the particles and that the clocks remain synchronized.

[a] e-mail: h.a.de.raedt@rug.nl

The set of data collected at station $i = 1, 2$ during an experiment with N detection events may be written as

$$\Upsilon_i = \{x_{n,i} = \pm 1, t_{n,i} | n = 1, \ldots, N\}, \qquad (1)$$

where n labels the event, $x_{n,i} = \pm 1$ specifies which of the two detectors fired and $t_{n,i}$ is the time-tag indicating the time at which a detector fired. In practice, the data $\{\Upsilon_1, \Upsilon_2\}$ may be analyzed for coincidences long after the data has been collected [9]. Coincidences are identified by comparing the time differences $\{t_{n,1} - t_{n,2} | n = 1, \ldots, N\}$ with a time window W [9]. Thus, for each pair of directions $\mathbf{a}_1$ and $\mathbf{a}_2$ of the Stern-Gerlach magnets, the number of coincidences $C_{xy} \equiv C_{xy}(\mathbf{a}_1, \mathbf{a}_2)$ between detectors $D_{x,1}$ ($x = \pm 1$) at station 1 and detectors $D_{y,2}$ ($y = \pm 1$) at station 2 is given by

$$C_{xy} = \sum_{n=1}^{N} \delta_{x,x_{n,1}} \delta_{y,x_{n,2}} \Theta(W - |t_{n,1} - t_{n,2}|), \qquad (2)$$

where $\Theta(t)$ is the Heaviside step function. We emphasize that we count all events that, according to the same criterion as the one employed in experiment, correspond to the detection of pairs. The correlation $E(\mathbf{a}_1, \mathbf{a}_2)$ between the data at station 1 and 2 is defined by

$$E(\mathbf{a}_1, \mathbf{a}_2) = \frac{C_{++} + C_{--} - C_{+-} - C_{-+}}{C_{++} + C_{--} + C_{+-} + C_{-+}}, \qquad (3)$$

where the denominator is the sum of all coincidences.

As is well-known, quantum theory itself has nothing to say about the individual events (quantum measurement paradox), but it provides a framework to compute the outcome of many repeated measurements [13,14]. The quantum mechanical description of the EPRB experiment assumes that the state of the two spin-1/2 particles is described by the singlet state $|\Psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$, with single-spin expectation values $\langle\Psi|\sigma_1 \cdot \mathbf{a}_1|\Psi\rangle = \langle\Psi|\sigma_2 \cdot \mathbf{a}_2|\Psi\rangle = 0$, and the two-spin correlation

$$\widehat{E}(\mathbf{a}_1, \mathbf{a}_2) = \langle\Psi|\sigma_1 \cdot \mathbf{a}_1 \; \sigma_2 \cdot \mathbf{a}_2|\Psi\rangle = -\mathbf{a}_1 \cdot \mathbf{a}_2. \qquad (4)$$

Here $\sigma_i = (\sigma_i^x, \sigma_i^y, \sigma_i^z)$ are the Pauli spin-1/2 matrices describing the spin of particle $i = 1, 2$ [14]. We introduced the notation $\widehat{\ }$ to distinguish the quantum mechanical results from the results obtained by analysis of the data $\{\Upsilon_1, \Upsilon_2\}$. Introducing the function

$$S(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}) = E(\mathbf{a}, \mathbf{c}) - E(\mathbf{a}, \mathbf{d}) + E(\mathbf{b}, \mathbf{c}) + E(\mathbf{b}, \mathbf{d}), \qquad (5)$$

it can be shown that $|\widehat{S}(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d})| \leq 2\sqrt{2}$, for any choice of the state $|\Psi\rangle$ [15]. On the other hand, $|\widehat{S}(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d})| \leq 2$, if $|\Psi\rangle$ is an uncorrelated state. In other words, if $S_{max} \equiv \max_{\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}} S(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}) > 2$ the system is in a correlated state. Thus, S_{max} is a convenient number to differentiate between uncorrelated and correlated states.

Analysis of the experimental data according to the procedure discussed earlier, demonstrates that $E(\mathbf{a}_1, \mathbf{a}_2) \approx \widehat{E}(\mathbf{a}_1, \mathbf{a}_2)$ [4–11], leading to the conclusion that in a quantum mechanical description, the state of the two spin-1/2 particles is correlated, even though the particles are spatially and temporally separated and do not interact.

We now propose a computer simulation model that generates the data $\{\Upsilon_1, \Upsilon_2\}$, see equation (1). The source emits particles that carry a three-dimensional unit vector $\mathbf{S}_{n,i} = (-1)^{i+1}(\cos\varphi_n \sin\theta_n, \sin\varphi_n \sin\theta_n, \cos\theta_n)$, representing the spin of the particles. The spin of a particle is completely characterized by φ_n and $\cos\theta_n$, which we assume to be distributed uniformly over the interval $[0, 2\pi[$ and $[-1, 1]$, respectively. The Stern-Gerlach magnet projects the spin $\mathbf{S}_{n,i}$ onto its axis of orientation $\mathbf{a}_i$ and deflects the particle according to the sign of the projection. Hence, $x_{n,i} = \mathrm{sign}(\mathbf{S}_{n,i} \cdot \mathbf{a}_i)$. To assign a time-tag to each event, we assume that as a particle passes through the detection system, it may experience a time delay $t_{n,i}$ which is distributed uniformly over the interval $[t_0, t_0 + T_{n,i}]$. From equation (2), it follows that only differences of time delays matter. Hence, we may put $t_0 = 0$. The time-tag for the event n is then $t_{n,i} \in [0, T_{n,i}]$. We thus need an explicit expression for $T_{n,i}$. The choice $T_{n,i} = \text{const.}$ is too simple: in this case we recover the model considered by Bell, which is known not to reproduce the correct quantum correlation equation (4) [16]. Assuming that the particle "knows" the direction of its own spin and the direction of the Stern-Gerlach magnet only, we can construct a number that is rotationally invariant: $\mathbf{S}_{n,i} \cdot \mathbf{a}_i$. Thus, we may assume $T_{n,i} = F(\mathbf{S}_{n,i} \cdot \mathbf{a}_i)$. As we already used $\mathbf{S}_{n,i} \cdot \mathbf{a}_i$ to determine whether the particle generates a $+1$ or -1 signal, it is not unreasonable to expect that F is a function of the outer product of both vectors and hence we choose, $T_{n,i} = T_0|\mathbf{S}_{n,i} \times \mathbf{a}_i|^d = T_0|1 - (\mathbf{S}_{n,i} \cdot \mathbf{a}_i)^2|^{d/2}$. Here, T_0 is the maximum time delay which defines the unit of time and d is a free parameter in our model. In the sequel, we express τ, W, $t_{n,i}$ and $T_{n,i}$ in units of T_0.

From the description of the model, it is evident that the data $x_{n,i}$ and $t_{n,i}$ recorded at station i depend on the variable $\mathbf{S}_{n,i}$ that the particles shared when they left the source and on the local variable $\mathbf{a}_i$, representing the orientation of the Stern-Gerlach magnet at station $i = 1, 2$. Furthermore, it is obvious that $x_{n,i}$ or $t_{n,i}$ do not depend on $\mathbf{a}_{3-i}$. This implies that for each event, the numbers $x_{n,i}$ and $t_{n,i}$ do not depend on whatever action is taken at observation station $3 - i$. Furthermore, the event n does not affect the data recorded for $n' \neq n$. Therefore, our model satisfies Einsteins condition of local causality. It should be noted that Einstein's concept of local causality is different from the condition of local causality introduced by Bell [16]. The former applies to every individual fact (ontological level), the latter merely to the probability of a fact to occur (epistemological level) [17]. The model that we propose does not rely on concepts of probability theory: it is a purely ontological model of the EPRB experiment. The coincidence equation (2), measured experimentally, cannot be written in terms of a product of two single-particle probabilities, an essential feature of the class of models examined by Bell [16]. Summarizing: our model trivially satisfies Einstein's criteria of local causality and realism.

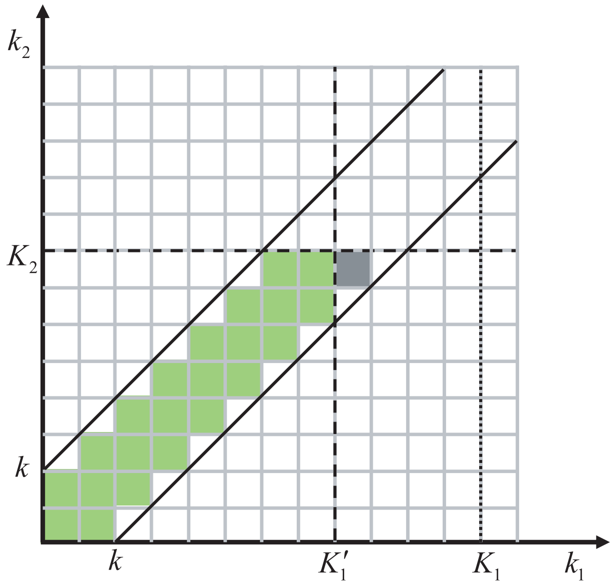

Fig. 2. (Color online) Graphical representation of the process of counting pairs. The time interval is divided in bins of size τ, represented by the elementary squares. The two parallel, 45° lines indicate the time window W, which was chosen to be 2τ in this example. In the limit $N \rightarrow \infty$, the total number of pairs for fixed $\mathbf{a}_i$ and (φ, θ) is given by the number of whole squares that fall within the time window and satisfy $1 \leq k_i < K_i$ for $i = 1, 2$. For $K_1 > K_2$, all filled squares contribute while for $K_1' = K_2$, the dark gray square does not contribute. For $K_1 < K_2$ we interchange labels 1 and 2.

Our model can easily be simulated on the computer, but for some choices of the parameters it can also be solved analytically. In the limit $N \rightarrow \infty$, equation (3) can be written as

$$E(\mathbf{a}_1, \mathbf{a}_2) = \frac{\int_0^{\pi} \int_0^{2\pi} x_1 x_2 P(T_1, T_2, W) \sin\theta d\theta d\varphi}{\int_0^{\pi} \int_0^{2\pi} P(T_1, T_2, W) \sin\theta d\theta d\varphi}, \quad (6)$$

where $P(T_1, T_2, W)$ is the density of coincidences for fixed $\mathbf{a}_i$ and angles (φ, θ) (within a small surface area $\sin\theta d\theta d\varphi$), $T_i \equiv F(\mathbf{S}_i \cdot \mathbf{a}_i)$, $\mathbf{S}_i = \mathbf{S}_i(\varphi, \theta)$ and $x_i = \text{sign}(\mathbf{S}_i \cdot \mathbf{a}_i)$.

An analytical expression for $P(T_1, T_2, W)$ can be derived as follows. For a fixed time-tag resolution $0 < \tau < 1$, the discretized time-tag for the nth detection event is given by $k_{n,i} = \lceil t_{n,i} \tau^{-1} \rceil$ where $\lceil x \rceil$ denotes the smallest integer that is larger or equal to x. The discretized time-tag $k_{n,i}$ takes integer values between 1 and $K_i \equiv \lceil \tau^{-1} T_i \rceil$, where K_i is the maximum, discretized time delay for a particle carrying angles (φ, θ) and passing through a Stern-Gerlach magnet with orientation $\mathbf{a}_i$. If $|k_{n,1} - k_{n,2}| < k = \lceil \tau^{-1} W \rceil$, the two spin-1/2 particles are defined to form a pair. For fixed $\mathbf{a}_i$ and (φ, θ), we can count the total number of pairs, or coincidences C, by considering the graphical representation shown in Figure 2. After a careful examination of all possibilities, we find that

$$\begin{aligned} C \equiv C(K_1, K_2, k) = (2k_0 - 1)k_{12} - k_0(k_0 - 1)/2 \\ - \max(0, (K_{12} - 1) \max(0, K_{12})/2) \\ + \max(0, k - k_0) k_0 \\ - \max(0, k k_{12} - K_1 K_2), \end{aligned} \quad (7)$$

where $k_0 = \min(K_1, K_2, k)$, $k_{12} = \min(K_1, K_2)$, and $K_{12} = k_{12} - \max(0, \max(K_1, K_2) - k)$.

It is clear that the result for the coincidences depends on the time-tag resolution τ, the time window W and the number of events N, just as in real experiments [4–11]. Equation (7) greatly simplifies if we consider the case $k = 1$ $(W = \tau)$, yielding $C(K_1, K_2, 1) = \min(K_1, K_2)$ as is evident by looking at Figure 2. For fixed $\mathbf{a}_i$, (φ, θ), and $W = \tau$, the density $P(T_1, T_2, \tau) = C(K_1, K_2, 1)/K_1 K_2$ that we register two particles with a time-tag difference less than τ is bounded by

$$\tau \frac{\min(T_1 + \tau, T_2 + \tau)}{(T_1 + \tau)(T_2 + \tau)} < P(T_1, T_2, \tau) \leq \tau \frac{\min(T_1, T_2)}{T_1 T_2}. \quad (8)$$

For $W = \tau \rightarrow 0$ and $T_i = |\mathbf{S}_i \times \mathbf{a}_i|^3$, the integrals in equation (6) can be evaluated in closed form. Without loss of generality, we can choose the coordinate system such that $\mathbf{a}_1 = (1, 0, 0)$ and $\mathbf{a}_2 = (\cos\alpha, \sin\alpha, 0)$. We find

$$E(\mathbf{a}_1, \mathbf{a}_2) = -\frac{\int_0^{2\pi} y_1 y_2 \frac{\min(\sin^2\varphi, \sin^2(\varphi - \alpha))}{\sin^2\varphi \sin^2(\varphi - \alpha)} d\varphi}{\int_0^{2\pi} \frac{\min(\sin^2\varphi, \sin^2(\varphi - \alpha))}{\sin^2\varphi \sin^2(\varphi - \alpha)} d\varphi}, \quad (9)$$

where $y_1 = \text{sign}(\cos\varphi)$ and $y_2 = \text{sign}(\cos(\varphi - \alpha))$. The integrals in equation (9) can be worked out analytically by careful examination of many different cases. The result of this somewhat tedious but straightforward exercise is $E(\mathbf{a}_1, \mathbf{a}_2) = -\mathbf{a}_1 \cdot \mathbf{a}_2$, which is exactly the same as the quantum mechanical result equation (4).

We now examine the case in which $W \rightarrow \infty$. Then, $\Theta(W - |t_{n,1} - t_{n,2}|) = 1$ and $P(T_1, T_2, W) = 1$ such that equation (6) reduces to [16]

$$E(\mathbf{a}_1, \mathbf{a}_2) = \frac{-1}{2\pi} \int_0^{\pi} \int_0^{2\pi} x_1 x_2 \sin\theta d\theta d\varphi = -1 + \frac{2|\alpha|}{\pi}. \quad (10)$$

Obviously, equation (10) does not agree with the quantum theoretical expression equation (4).

For $d = 3$, the model behavior changes from quantum mechanical correlated (entangled) to uncorrelated as W increases from zero to infinity. This can also be seen from Figure 3 which shows S_{max} as a function of W/τ for various values of d. S_{max} is calculated numerically using equations (6) and (7). Note that the numerical results agree with the values of S_{max} that have been obtained analytically for $W = \tau \rightarrow 0$, $d = 0, 3$ and $W \rightarrow \infty$. For $d < 3$, $2 \leq S_{max} < 2\sqrt{2}$ for any value of W/τ. Hence, for $d < 3$ our model cannot produce the correlations of the singlet state. For $d = 3$, $2 \leq S_{max} \leq 2\sqrt{2}$ and our model produces the correlations of the singlet state if $W/\tau \rightarrow 0$. For $d > 3$, $2 \leq S_{max} \leq 4$, and for a range of W/τ, $S_{max} > 2\sqrt{2}$, implying that our model exhibits correlations that cannot be

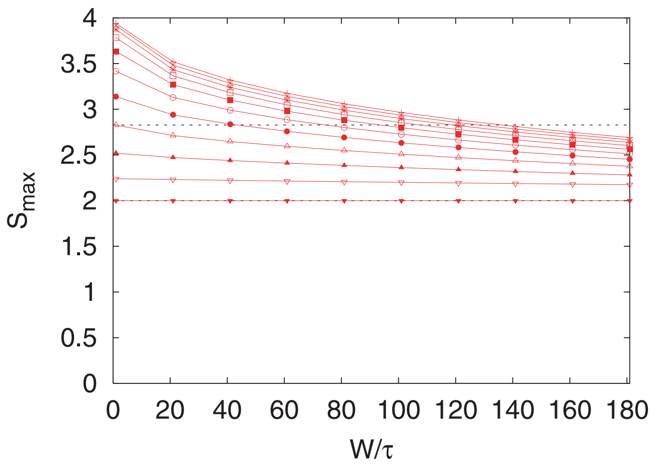

Fig. 3. (Color online) Maximum S_{max} of $S(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d})$ as a function of the time window W relative to the time-tag resolution τ. Curves from bottom to top: results for $d = 0, 1, \ldots, 10$. Dashed line: Value of $S_{max} = 2\sqrt{2}$ if the system is described by quantum theory.

Rapid Note

described by the quantum theory of two spin-1/2 particles, while it still satisfies Einstein's criteria for local causilty and realism.

In summary, starting from the factual observation that experimental realizations of the EPR *gedanken* experiment produce the data $\{\Upsilon_1, \Upsilon_2\}$ (see Eq. (1)) and that coincidence in time is a key ingredient for the data analysis, we have constructed a computer simulation model that satisfies Einstein's conditions of local causality and realism and exactly reproduces the correlation $E(\mathbf{a}_1, \mathbf{a}_2) = -\mathbf{a}_1 \cdot \mathbf{a}_2$ that is characteristic for a quantum system in the singlet state. Salient features of this model are that it generates the data set equation (1) event-by-event, uses integer arithmetic and elementary mathematics to analyze the data, does not rely on concepts of probability theory and quantum theory, and provides a simple, rational and realistic picture of the mechanism that yields correlations such as equation (4).

We have shown that whether or not our model produces quantum correlations depends on the data analysis procedure that is performed after the data has been collected: in order to observe the correlations of the singlet state, the resolution τ of the devices that generate the time-tags and the time window W should be made as small as possible. Disregarding the time-tag data ($d = 0$ or $W \to \infty$) yields results that disagree with quantum theory but agree with the models considered by Bell [16]. The transition from a correlated (entangled) state to an uncorrelated state can be easily checked without repeating the experiments. Indeed, according to our model, the existence of such a transition can be tested by re-analyzing available experimental data with different values of the time window W.

Although quantum theory provides the framework to compute the frequencies for observing events, it does not describe individual events themselves [13,14]. Our work suggests that it is possible to construct event-based simulation models that satisfies Einstein's criteria of local causality and realism and can reproduce the expectation values calculated by quantum theory [18–22]. It therefore opens new routes to ontological descriptions of microscopic phenomena.

References

1. A. Einstein, A. Podolsky, N. Rosen, Phys. Rev. **47**, 777 (1935)
2. D. Bohm, *Quantum Theory* (Prentice-Hall, New York, 1951)
3. W. Gerlach, O. Stern, Z. Phys. **9**, 349 (1922)
4. S.J. Freedman, J.F. Clauser, Phys. Rev. Lett. **28**, 938 (1972)
5. A. Aspect, P. Grangier, G. Roger, Phys. Rev. Lett. **49**, 91 (1982)
6. A. Aspect, J. Dalibard, G. Roger, Phys. Rev. Lett. **49**, 1804 (1982)
7. P.R. Tapster, J.G. Rarity, P.C.M. Owens, Phys. Rev. Lett. **73**, 1923 (1994)
8. W. Tittel, J. Brendel, H. Zbinden, N. Gisin, Phys. Rev. Lett. **81**, 3563 (1998)
9. G. Weihs, T. Jennewein, C. Simon, H. Weinfurther, A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1998)
10. M.A. Rowe, D. Kielpinski, V. Meyer, C.A. Sackett, W.M. Itano, C. Monroe, D.J. Wineland, Nature **401**, 791 (2001)
11. D. Fatal, E. Diamanti, K. Inoue, Y. Yamamoto, Phys. Rev. Lett. **92**, 037904 (2004)
12. J.F. Clauser, M.A. Horne, Phys. Rev. D **10**, 526 (1974)
13. D. Home, *Conceptual Foundations of Quantum Physics* (Plenum Press, New York, 1997)
14. L.E. Ballentine, *Quantum Mechanics: A Modern Development* (World Scientific, Singapore, 2003)
15. B.S. Cirel'son, Lett. Math. Phys. **4**, 93 (1980)
16. J.S. Bell, *Speakable and unspeakable in quantum mechanics* (Cambridge University Press, Cambridge, 1993)
17. E.T. Jaynes, *Clearing up mysteries - The original goal*, in *Maximum Entropy and Bayesian Methods*, edited by J. Skilling (Kluwer Academic Publishers, Dordrecht, 1989)
18. K. De Raedt, H. De Raedt, K. Michielsen, Comp. Phys. Comm. **171**, 19 (2005)
19. H. De Raedt, K. De Raedt, K. Michielsen, J. Phys. Soc. Jpn. Suppl. **76**, 16 (2005)
20. H. De Raedt, K. De Raedt, K. Michielsen, Europhys. Lett. **69**, 861 (2005)
21. K. Michielsen, K. De Raedt, H. De Raedt, J. Comput. Theor. Nanosci. **2**, 227 (2005)
22. H. De Raedt, K. De Raedt, K. Michielsen, S. Miyashita, Comp. Phys. Comm. **174**, 803 (2006)