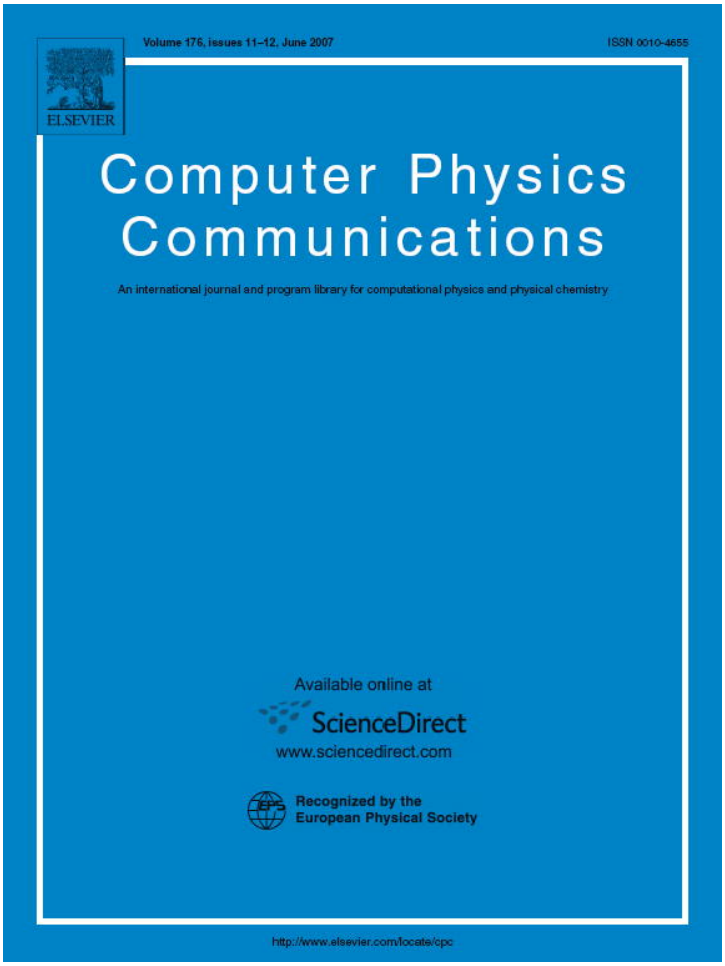

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A computer program to simulate Einstein–Podolsky–Rosen–Bohm experiments with photons

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Abstract

Starting from the data gathering and analysis procedures used in Einstein–Podolsky–Rosen–Bohm experiments with photons, we construct a simulation algorithm that satisfies Einstein's criteria of local causality and realism and generates the same type of data as recorded in these idealized experiments. The simulation data is analyzed according to the experimental procedure to count coincidences, that is by using an expression for the coincidence counts that, besides depending on the settings of the variable polarizers, explicitly depends on the difference of the time tags of the detection signals in both observation stations and on an adjustable time window. We demonstrate that the simulation algorithm produces data that agrees with the two-particle correlation for the singlet state.

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Keywords: Quantum mechanics; Computational techniques

1. Introduction

Computer simulation is a powerful methodology to study physical phenomena and is widely regarded as complementary to theory and experiment [1]. The computer simulation approach usually builds on the basic equations of physics and employs numerical algorithms to solve these equations. At present, there are only a few physical phenomena that cannot be simulated on a computer.

If the phenomena are microscopic and concern individual events rather than the statistical properties of many events, the theory that describe the latter, quantum theory, is of no avail. Indeed, as is well known from the early days in the development of quantum theory, quantum theory has nothing to say about individual events [2–4]. Reconciling the mathematical formalism of quantum theory with the experimental fact that each observation yields a definite outcome is referred to as the quantum measurement paradox. It is the most fundamental problem in the foundation of quantum theory [3].

It is well known that the analysis of the Einstein–Podolsky–Rosen (EPR) *gedanken* experiment leads to the conclusion that quantum theory is not a complete theory [5]. The fundamental problem, exposed by the EPR *gedanken* experiment, is to explain how individual events, registered by different detectors in such a way that a measurement on one particle does not have a causal effect on the result of the measurement on another particle (Einstein's criterion of local causality), exhibit two-particle quantum correlations that are found in experiments [6–13]. In the absence of a theory that describes the individual events, the very successful computational-physics approach "start from the theory and invent/use a simulation algorithm" cannot be applied to this problem.

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If computer simulation is indeed a third methodology to study physical phenomena, it should be possible to simulate real EPR experiments event by event. The basic idea behind this is simple. The result of any such experiment is a collection of numbers. If we succeed to construct an algorithm, that is an arithmetically well-defined procedure, that generates the same kind of numbers as obtained in these experiments and, after applying the same data analysis as the one employed by the experimenters, leads us to draw the same conclusions then we have found an algorithm that simulates the phenomenon. Within the context of probabilistic models, a rigorous proof that it is not impossible to describe real EPRB experiments has already been given [14].

Recently, we introduced an Einstein-local model for correlations of the singlet state and gave a rigorous proof that this model reproduces the results of quantum theory for this state [15]. In this paper, we use this model to perform the first, complete Einstein-local computer simulation of Aspect-type experiments. We demonstrate that it is possible to reproduce the results of quantum theory by simulating real EPR experiments in exactly the same manner as the experiments and the subsequent data analysis are carried out. We take as an example, the EPR–Bohm (EPRB) experiment with photons performed by Weihs et al. [10], an experiment that is the successor of the timing experiment with optical switches by Aspect et al. [7] and that comes very close to realizing the EPR *gedanken* experiment.

An important feature in these experiments is the arbitrariness in the choice of the directions in which the polarization will be measured, for each individual detection event [6–10,13]. This feature has not been taken into account in the rigorous proof [15] (and in quantum theory) but is fully accounted for in the simulation procedure that we describe in this paper. We simulate ideal experiments only. Hence, we do not consider practical loopholes, such as detection problems (lost photons and registration of accidental signals) and synchronization problems, that might be present in real experiments.

According to Bell's theorem, as it is commonly formulated, an algorithm such as the one that we describe in this paper is not supposed to exist. Indeed, Bell's theorem states that any local, hidden variable model will produce results that are in conflict with the quantum theory of a system of two $S = 1/2$ particles [16]. However, it is often overlooked that this statement can be proven for a (very) restricted class of probabilistic models only. Indeed, minor modifications to the original model of Bell lead to the conclusion that there is no conflict [14,17,18]. Furthermore, it is easy to see that there can be no real conflict because simulation algorithms and actual data do not necessarily satisfy the (hidden) conditions under which Bell's theorem hold [19–21].

It is important to recognize that there is a significant conceptual difference between the arithmetic rules by which the artificial dynamical system (the digital computer) evolves and the mathematical model, quantum theory, the results of which we would like to reproduce. The standard interpretation of quantum theory uses concepts of probability theory to relate the objects in the mathematical theory (expectation values) to the distribution of events (Bernoulli-trials) [4]. In this paper, we confine the discussion to simulation algorithms that run on a digital computer. Then, the time evolution of the artificial dynamical system is, in principle, strictly deterministic. Thus, in this artificial dynamical system, there are no "random variables" in the strict mathematical sense.

This conceptual difference becomes most evident by considering a simple example, namely the pseudo-random number generators that are commonly used in simulation work [1]. If we know the seed of the pseudo-random number generator, as we usually do, we can predict the time series of the pseudo-random number with absolute certainty [1]. Thus, these numbers are no "random variables" in the strict mathematical sense. Probability theory has nothing useful to say about the deterministic sequence of these numbers. In fact, it does not even contain, nor provides a procedure to generate random variables. The latter is a very useful, mental construct but has no counterpart in the realm of algorithms. However, once we start computing averages and correlations of the time series and the pseudo-random number generator is of sufficient quality [1], we usually conclude that *for the purpose that we intend to use these numbers for*, the pseudo-random numbers have properties similar to those of random variables, a well-defined mathematical concept in probability theory only. Once we accept this conclusion as a working hypothesis, we can make contact to probability theory and use the machinery of the latter to analyze the problem further.

Summarizing: A priori, a probabilistic theory has nothing to say about the results of an algorithm that runs on a digital computer. It may happen that some properties of the data, generated by an algorithm, or data recorded in an experiment, can be described very well by a probabilistic model. The justification for adopting the probabilistic model as a vehicle to describe the data comes a posteriori, that is if the agreement between simulation data and results obtained from the mathematical model is considered to be satisfactory.

The fact that there exist data (just plain numbers stored on a computer disk) that is collected during an experiment and analyzed long after the data is taken yields expectation values that are in concert with quantum theory provides a strong indication that there may be algorithms that generate the same kind of data and reproduce the results of quantum theory. Indeed, if the type-of-data/procedure in the simulation and experiment are exactly the same, then why would it be "allowed" for the experimental data (just a collection of bytes on a hard disk) to yield the correlation of the singlet state and would it not be allowed for a simulation model that produces the same kind of bytes to do the same? Of course, in order to find such an algorithm, it is essential that it generates the same kind of data items as those recorded and analyzed in the experiment. Therefore we should not restrict the search for such an algorithm to the class that generates less data items (see later), as is done in all textbook treatments of EPR experiments.

In this paper, we confine ourselves to a description of the simulation algorithm, its implementation, and a discussion of some simulation results. The implications of our work on the interpretation of quantum theory and other, more philosophical matters will be discussed elsewhere.

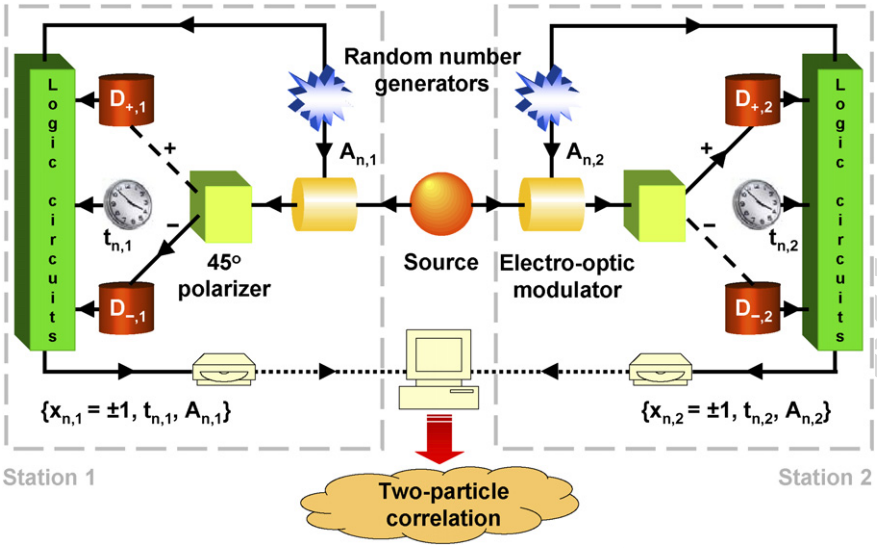

Fig. 1. Schematic diagram of an EPRB experiment with photons [10].

2. EPRB experiment with photons

A schematic diagram of the experiment is shown in Fig. 1 (see also Fig. 2 in [10]). A source emits pairs of photons with opposite polarization. Each photon of a pair propagates to an observation station in which it is manipulated and detected. The two stations are separated spatially and temporally [10]. This arrangement prevents the observation at station 1 (2) to have a causal effect on the data registered at station 2 (1) [10].

As the photon arrives at station $i = 1, 2$, it passes through an electro-optic modulator that rotates the polarization of the photon by an angle depending on the voltage applied to the modulator. These voltages are controlled by two independent binary random number generators. As the photon leaves the polarizer, it generates a signal in one of the two detectors. The station's clock assigns a time-tag to each generated signal. Effectively, this procedure discretizes time in intervals of a width that is determined by the time-tag resolution τ [10].

In the experiment, the firing of a detector is regarded as an event. At the nth event, the data recorded on a hard disk at station $i = 1, 2$ consists of $x_{n,i} = \pm 1$, specifying which of the two detectors fired, the time tag $t_{n,i}$ indicating the time at which a detector fired, and the random number $A_{n,i} = 1, 2$ that determines the voltage that is applied to the electro-optic modulator [10]. There is a one-to-one correspondence between the random number $A_{n,i}$ and the angle $\gamma_{n,i}$ of the rotation of the polarization [10]. Hence, the set of data collected at station $i = 1, 2$ during a run of N events may be written as

$$\Upsilon_i = \{x_{n,i} = \pm 1, t_{n,i}, \gamma_{n,i} \mid n = 1, \ldots, N\}. \tag{1}$$

The crucial point is that in any real EPRB-type experiment, it is necessary to have an operational procedure to decide if the two detection events correspond to the observation of one two-particle system or to the observation of two single-particle systems. In standard "hidden variable" treatments of the EPR gedanken experiment, the operational definition of "observation of a single two-particle system" is missing. In EPRB-type experiments, this decision is taken on the basis of coincidence in time [10,22]. Coincidences are identified by comparing the time differences $\{t_{n,1} - t_{n,2} \mid n = 1, \ldots, N\}$ with a time window W [10]. Thus, for each pair of rotation angles α and β, the number of coincidences between detectors $D_{x,1}$ $(x = \pm 1)$ at station 1 and detectors $D_{y,2}$ $(y = \pm 1)$ at station 2 is given by

$$C_{xy} = C_{xy}(\alpha, \beta) = \sum_{n=1}^{N} \delta_{x,x_{n,1}} \delta_{y,x_{n,2}} \delta_{\alpha,\gamma_{n,1}} \delta_{\beta,\gamma_{n,2}} \Theta\big(W - |t_{n,1} - t_{n,2}|\big), \tag{2}$$

where $\Theta(t)$ is the Heaviside step function. The correlation $E(\alpha, \beta)$ between the coincidence counts is defined by

$$E(\alpha, \beta) = \frac{C_{++} + C_{--} - C_{+-} - C_{-+}}{C_{++} + C_{--} + C_{+-} + C_{-+}} = \frac{\sum_{x,y=\pm 1} xy C_{xy}}{\sum_{x,y=\pm 1} C_{xy}}, \tag{3}$$

where the denominator in Eq. (3) is the sum of all coincidences [10]. In practice, the data sets $\{\Upsilon_1, \Upsilon_2\}$ are analyzed long after the data has been collected [10]. In general, the values for the coincidences $C_{xy}(\alpha, \beta)$ and correlation $E(\alpha, \beta)$ depend on the time-tag resolution τ and the time window W used to identify the coincidences.

All textbook treatments of the EPRB experiment assume that the correlation, as measured in experiment, is given by [16]

$$C_{xy}^{(\infty)} = \sum_{n=1}^{N} \delta_{x,x_{n,1}} \delta_{y,x_{n,2}}, \tag{4}$$

which is obtained from Eq. (2) by taking the limit $W \to \infty$, hence the notation $C_{xy}^{(\infty)}$.

Although $C_{xy}^{(\infty)}$ defines a valid theoretical model, there is no reason why this model should have any bearing on the real experiments, in particular because in experiments a lot of effort is made to reduce (not increase) W [10,23]. We have already shown that for $W \to 0$, it is relatively easy to reproduce the experimental facts and the results of quantum theory [15]. Furthermore, keeping W arbitrary does not render the mathematics more complicated so there really is no point of studying the simplified model defined by Eq. (4) separately: We may always consider the limiting case $W \to \infty$ afterwards.

3. Quantum theory

According to the axioms of quantum theory [4], repeated measurements on the two-spin system described by the density matrix ρ yield statistical estimates for the single-spin expectation values

$$\widetilde{E}_1(\mathbf{a}) = \langle \boldsymbol{\sigma}_1 \cdot \mathbf{a} \rangle, \qquad \widetilde{E}_2(\mathbf{b}) = \langle \boldsymbol{\sigma}_2 \cdot \mathbf{b} \rangle, \tag{5}$$

and the two-spin expectation value

$$\widetilde{E}(\mathbf{a}, \mathbf{b}) = \langle \boldsymbol{\sigma}_1 \cdot \mathbf{a}\, \boldsymbol{\sigma}_2 \cdot \mathbf{b} \rangle, \tag{6}$$

where $\boldsymbol{\sigma}_i = (\sigma_i^x, \sigma_i^y, \sigma_i^z)$ are the Pauli spin-1/2 matrices describing the spin of particle $i = 1, 2$ [4], and $\mathbf{a}$ and $\mathbf{b}$ are unit vectors. We have introduced the tilde to distinguish the quantum mechanical results from the results obtained from the data sets $\{\Upsilon_1, \Upsilon_2\}$. The state of a quantum system of two $S = 1/2$ objects is completely determined if we know the expectation values $\widetilde{E}_1(\mathbf{a})$, $\widetilde{E}_2(\mathbf{b})$, and $\widetilde{E}(\mathbf{a}, \mathbf{b})$.

Introducing the function

$$S(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}) = E(\mathbf{a}, \mathbf{c}) - E(\mathbf{a}, \mathbf{d}) + E(\mathbf{b}, \mathbf{c}) + E(\mathbf{b}, \mathbf{d}), \tag{7}$$

it can be shown that $|\widetilde{S}(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d})| \leqslant 2\sqrt{2}$ [24], independent of the choice of ρ. If the density matrix $\rho = \rho_1 \otimes \rho_2$ factorizes (here ρ_i is the 2×2 density matrix of spin i), then it is easy to prove that $|\widetilde{S}(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d})| \leqslant 2$. In other words, if $\max_{\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}} \widetilde{S}(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}) > 2$, then $\rho \neq \rho_1 \otimes \rho_2$, and the quantum system is in an entangled state. Specializing to the case of the photon polarization, the unit vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$ lie in the same plane and we may use the angles α, α', β, and β' to specify their direction.

The quantum theoretical description of the EPRB experiment assumes that the system is represented by the singlet state $|\Psi\rangle = (|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2)/\sqrt{2}$ of two spin-1/2 particles, where H and V denote the horizontal and vertical polarization and the subscripts refer to photons 1 and 2, respectively. For the singlet state $\rho = |\Psi\rangle\langle\Psi|$,

$$\widetilde{E}_1(\alpha) = \widetilde{E}_2(\beta) = 0, \tag{8}$$

$$\widetilde{E}(\alpha, \beta) = -\cos 2(\alpha - \beta), \tag{9}$$

for which $\max_{\alpha,\alpha',\beta,\beta'} \widetilde{S}(\alpha, \alpha', \beta, \beta') = 2\sqrt{2}$, confirming that the singlet is a quantum state with maximal entanglement.

Analysis of the experimental data according to the procedure sketched earlier [6–12], yields results that are in good agreement with $\widetilde{E}_1(\alpha) = \widetilde{E}_2(\beta) = 0$ and $\widetilde{E}(\alpha, \beta) = -\cos 2(\alpha - \beta)$, leading to the conclusion that in a quantum mechanical description, the density matrix does not factorize, in spite of the fact that the photons are spatially and temporally separated and do not interact.

3.1. Relation between quantum theory and experiment

Let us now try to relate the quantum theoretical expectation values that appear in Eqs. (5) and (6) to the actual data. In general, the probability for observing a pair of dichotomic variables $\{x, y\}$ can be written as

$$\widehat{P}(x, y|\mathbf{a}, \mathbf{b}) = \frac{1 + x\widehat{E}_x(\mathbf{a}, \mathbf{b}) + y\widehat{E}_y(\mathbf{a}, \mathbf{b}) + xy\widehat{E}_{xy}(\mathbf{a}, \mathbf{b})}{4}, \tag{10}$$

from which, by the standard rules of probability theory, it follows that

$$\widehat{P}_y(x|\mathbf{a}, \mathbf{b}) \equiv \sum_{y=\pm 1} \widehat{P}(x, y|\mathbf{a}, \mathbf{b}) = \frac{1 + x\widehat{E}_x(\mathbf{a}, \mathbf{b})}{2}, \tag{11}$$

$$\widehat{P}_x(y|\mathbf{a}, \mathbf{b}) \equiv \sum_{x=\pm 1} \widehat{P}(x, y|\mathbf{a}, \mathbf{b}) = \frac{1 + y\widehat{E}_y(\mathbf{a}, \mathbf{b})}{2}. \tag{12}$$

By definition, x and y are logically independent if and only if $P(x, y|\mathbf{a}, \mathbf{b}) = P_y(x|\mathbf{a}, \mathbf{b})P_x(y|\mathbf{a}, \mathbf{b})$ [4,25,26]. If x and y are logically independent it is easy to show that $E_{xy} = E_x E_y$. In general, the converse is not true [4,25,26] but in this particular case it is. Indeed, if $E_{xy} = E_x E_y$, if follows directly from Eq. (10) that $P(x, y|\mathbf{a}, \mathbf{b}) = P_y(x|\mathbf{a}, \mathbf{b})P_x(y|\mathbf{a}, \mathbf{b})$. Thus $E_{xy} \neq E_x E_y$ if and only if x and y are logically dependent.

In quantum theory, we also have two different cases. If the density matrix of the two $S = 1/2$ quantum system factorizes, we have $\langle \boldsymbol{\sigma}_1 \cdot \mathbf{a}\, \boldsymbol{\sigma}_2 \cdot \mathbf{b}\rangle = \langle \boldsymbol{\sigma}_1 \cdot \mathbf{a}\rangle\langle \boldsymbol{\sigma}_2 \cdot \mathbf{b}\rangle$ and the state of the system is completely characterized by $\widetilde{E}_1(\mathbf{a})$ and $\widetilde{E}_2(\mathbf{b})$. However, if the density matrix does not factorize, a complete characterization of this entangled state requires the knowledge of $\widetilde{E}_1(\mathbf{a})$, $\widetilde{E}_2(\mathbf{b})$, and $\widetilde{E}(\mathbf{a}, \mathbf{b})$. Upto this point, it seems that there is full analogy with the probabilistic model of the data, but we still have to relate the quantum theoretical expressions to the observed data.

To this end, we invoke the postulate that states that the possible values of a dynamical variable in quantum theory are the eigenvalues of the linear operator that corresponds to this variable [4]. For the case at hand, the operators are $\boldsymbol{\sigma}_1 \cdot \mathbf{a}$, $\boldsymbol{\sigma}_2 \cdot \mathbf{b}$, and $\boldsymbol{\sigma}_1 \cdot \mathbf{a}\, \boldsymbol{\sigma}_2 \cdot \mathbf{b}$, with eigenvalues $\tilde{x} = \pm 1$, $\tilde{y} = \pm 1$ and $\tilde{z} = \pm 1$, respectively.

It is evident that the triples $\{\tilde{x}, \tilde{y}, \tilde{z}\}$ cannot represent the data equation (1) that is recorded and analyzed in real EPR experiments: The quantum mechanical model is trivially incomplete in that it has no means to describe the time-tag data. But, quantum theory is incomplete in a more fundamental sense [2,5].

First, let us consider an experiment that produces z only. A consistent application of the postulates of quantum theory yields

$$P(\tilde{z}|\mathbf{a}, \mathbf{b}) = \frac{1 + \tilde{z}\widetilde{E}(\mathbf{a}, \mathbf{b})}{2}, \tag{13}$$

and we would use $\widetilde{E}(\mathbf{a}, \mathbf{b}) = E_{xy}(\mathbf{a}, \mathbf{b})$ to relate the theoretical result to the data. Likewise, we could imagine an experiment that produces x (y) and use $\widetilde{E}_1(\mathbf{a}) = E_x(\mathbf{a}, \mathbf{b})$ ($\widetilde{E}_2(\mathbf{b}) = E_y(\mathbf{a}, \mathbf{b})$) to relate the theoretical description to the data.

Second, we imagine an experiment that yields the data $\{x, y\}$ and we ask whether it is possible to describe the outcome of this experiment by quantum theory. According to the postulates of quantum theory, the probabilities for the eigenvalues to take the values $\{\tilde{x}, \tilde{y}\}$ are given by

$$P(\tilde{x}|\mathbf{a}, \mathbf{b}) = \frac{1 + \tilde{x}\widetilde{E}_1(\mathbf{a})}{2}, \tag{14}$$

$$P(\tilde{y}|\mathbf{a}, \mathbf{b}) = \frac{1 + \tilde{y}\widetilde{E}_2(\mathbf{b})}{2}, \tag{15}$$

where $\tilde{x}$, and $\tilde{y}$ are logically independent random variables, that is each measurement of a dynamical variable constitutes a Bernoulli trial [4]. Then, we would use $\widetilde{E}_1(\mathbf{a}) = E_x(\mathbf{a}, \mathbf{b})$ and $\widetilde{E}_2(\mathbf{b}) = E_y(\mathbf{a}, \mathbf{b})$ to relate the theory to the data.

But the real data is $\{x, y\}$, not the logically independent random variables $\{\tilde{x}, \tilde{y}\}$ of the mathematical model. Therefore the quantum mechanical description of an experiment that yields $\{x, y\}$ is necessarily incomplete if the data is such that $E_{xy} \neq E_x E_y$.

The fact that EPRB experiments show good agreement with the quantum theory of two $S = 1/2$ objects is not in conflict with this reasoning: In real EPRB experiments, the coincidences are computed according to Eq. (2), which includes the time-tag information, about which quantum theory has nothing to say. Hence there is no logical inconsistency.

4. Simulation model

A concrete simulation model of the EPRB experiment sketched in Fig. 1 requires a specification of the information carried by the particles, of the algorithm that simulates the source and the observation stations, and of the procedure to analyze the data. In the following, we describe a slightly modified version of the algorithm proposed in Ref. [15], tailored to the case of photon polarization.

Source and particles. The source emits particles that carry a vector $\mathbf{S}_{n,i} = (\cos(\xi_n + (i-1)\pi/2), \sin(\xi_n + (i-1)\pi/2))$, representing the polarization of the photons that travel to station $i = 1$ and station $i = 2$, respectively. Note that $\mathbf{S}_{n,1} \cdot \mathbf{S}_{n,2} = 0$, indicating that the two particles have orthogonal polarizations. The "polarization state" of a particle is completely characterized by ξ_n, which is distributed uniformly over the whole interval $[0, 2\pi[$. For this purpose, to mimic the apparent unpredictability of the experimental data, we use uniform random numbers. However, from the description of the algorithm, it will be clear that the use of random numbers is not essential. Simple counters that sample the interval $[0, 2\pi[$ in a systematic, but uniform, manner might be employed as well.

Observation station. The electro-optic modulator in station i rotates $\mathbf{S}_{n,i}$ by an angle $\gamma_{n,i}$. The number M of different rotation angles is chosen prior to the data collection (in the experiment of Weihs et al., $M = 2$ [10]). We use $2M$ random numbers to fill the arrays $(\alpha_1, \ldots, \alpha_M)$ and $(\beta_1, \ldots, \beta_M)$. During the measurement process we use two uniform random numbers $1 \leqslant m, m' \leqslant M$ to select the rotation angles $\gamma_{n,1} = \alpha_m$ and $\gamma_{n,2} = \beta_{m'}$. The electro-optic modulator then rotates $\mathbf{S}_{n,i} = (\cos(\xi_n + (i-1)\pi/2), \sin(\xi_n + (i-1)\pi/2))$ by $\gamma_{n,i}$, yielding $\mathbf{S}_{n,i} = (\cos(\xi_n - \gamma_{n,i} + (i-1)\pi/2), \sin(\xi_n - \gamma_{n,i} + (i-1)\pi/2))$.

The polarizer at station i projects the rotated vector onto its x-axis: $\mathbf{S}_{n,i} \cdot \hat{\mathbf{x}}_i = \cos(\xi_n - \gamma_{n,i} + (i-1)\pi/2)$, where $\hat{\mathbf{x}}_i$ denotes the unit vector along the x-axis of the polarizer. For the polarizing beam splitter, we consider a simple model: If $\cos^2(\xi_n - \gamma_{n,i} + (i-1)\pi/2) > 1/2$ the particle causes $D_{+1,i}$ to fire, otherwise $D_{-1,i}$ fires. Thus, the detection of the particles generates the data $x_{n,i} = \text{sign}(\cos 2(\xi_n - \gamma_{n,i} + (i-1)\pi/2))$.

Time-tag model. To assign a time-tag to each event, we assume that as a particle passes through the detection system, it may experience a time delay. According to Maxwell's equation, in the optically anisotropic materials used to fabricate polarizers or electro-optic modulators, plane waves with different polarization propagate with different velocity and are refracted differently [27]. Although there is some experimental data on the time delay caused by the electro-optic modulator [23], a systematic study of this aspect is missing. In our model, the time delay $t_{n,i}$ for a particle is assumed to be distributed uniformly over the interval $[t_0, t_0 + T]$, an assumption that is not in conflict with available data [23]. In practice, we use uniform random numbers to generate $t_{n,i}$. As in the case of the angles ξ_n, the random choice of $t_{n,i}$ is merely convenient, not essential. From Eq. (2), it follows that only differences of time delays matter. Hence, we may put $t_0 = 0$. The time-tag for the event n is then $t_{n,i} \in [0, T]$.

We now come to the point that we have to specify T explicitly. In fact, there are not many options. Assuming that the particle "knows" its own direction and that of the polarizer only, we can construct one number that depends on the relative angle: $\mathbf{S}_{n,i} \cdot \hat{\mathbf{x}}_i$. Thus, $T = T(\xi_n - \gamma_{n,i})$ depends on $\xi_n - \gamma_{n,i}$ only. Furthermore, consistency with classical electrodynamics requires that functions that depend on the polarization have period π [27]. Thus, we must have $T(\xi_n - \gamma_{n,i} + (i-1)\pi/2) = F((\mathbf{S}_{n,i} \cdot \hat{\mathbf{x}}_i)^2)$. We already used $\cos 2(\xi_n - \gamma_{n,i} + (i-1)\pi/2)$ to determine whether the particle generates a $+1$ or -1 signal. Hence, it is not unreasonable to expect that $T(\xi_n - \gamma_{n,i} + (i-1)\pi/2) = T_0(\sin 2(\xi_n - \gamma_{n,i} + (i-1)\pi/2))^d$ yields useful results. Here, $T_0 = \max_\theta T(\theta)$ is the maximum time delay and defines the unit of time.

Data analysis. For fixed N and M, the algorithm generates the data sets Υ_i just as experiment does [10]. In order to count the coincidences, we choose a time-tag resolution $0 < \tau < T_0$ and a coincidence window $\tau \leqslant W$. We set the correlation counts $C_{xy}(\alpha_m, \beta_{m'})$ to zero for all $x, y = \pm 1$ and $m, m' = 1, \ldots, M$. We compute the discretized time tags $k_{n,i} = \lceil t_{n,i}/\tau \rceil$ for all events in both data sets. Here $\lceil x \rceil$ denotes the smallest integer that is larger or equal to x, that is $\lceil x \rceil - 1 < x \leqslant \lceil x \rceil$. According to the procedure adopted in the experiment [10], an entangled photon pair is observed if and only if $|k_{n,1} - k_{n,2}| < k = \lceil W/\tau \rceil$. Thus, if $|k_{n,1} - k_{n,2}| < k$, we increment the count $C_{x_{n,1},x_{n,2}}(\alpha_m, \beta_{m'})$.

4.1. Implementation

In Appendix A, we list a demo FORTRAN 90 program that implements the simulation algorithm. We believe that the reader should have no difficulties identifying the variables and pieces of code with those used in the text above. We have tried to keep the source code simple, the most "complicated" part being the section that prints out the results. We have omitted the code that reshuffles the randomly chosen angles $(\alpha_1, \ldots, \alpha_M)$ and $(\beta_1, \ldots, \beta_M)$ such that the angle difference lies in the interval $[0, \pi]$, as in Fig. 2. The program computes the correlation $E(\alpha_i, \beta_j)$, with and without using the time-tag information, as well as the corresponding single-detection counts $E_1(\alpha_i)$ and $E_2(\beta_j)$.

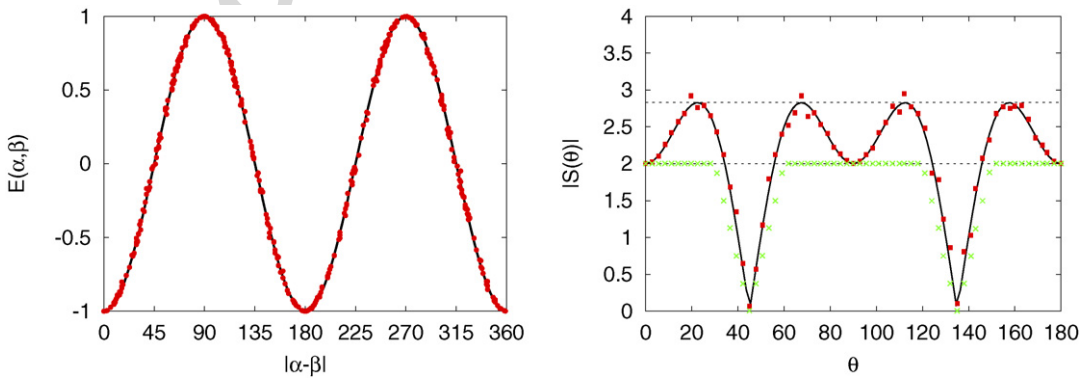

Fig. 2. Left: Comparison between computer simulation data (red bullets) and quantum theory (black solid line) for the two-particle correlation $E(\alpha, \beta)$. Right: Computer simulation results for $S(\theta)$. Squares (red): Analysis using all the data of the sets $\{\Upsilon_1, \Upsilon_2\}$; Crosses (green): Analysis without using the time-tag data to compute the two-particle correlation. Solid line (black): Quantum theory ($\widetilde{S}(\theta) = \cos 6\theta - 3\cos\theta$). Dashed line at $S(\theta) = 2\sqrt{2}$: Maximum of $\widetilde{S}(\theta)$. Dashed line at $S(\theta) = 2$: Maximum of $S(\theta)$ if the system is described by the class of models introduced by Bell [16]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

5. Simulation results

The simulation proceeds in the same way as the experiment, that is we first collect the data sets $\{\Upsilon_1, \Upsilon_2\}$, and then compute the coincidences (see Eq. (2)) and the correlation (see Eq. (3)). The simulation results for the coincidences $C_{xy}(\alpha, \beta)$ depend on the time-tag resolution τ, the time window W and the number of events N, just as in real experiments. In the left panel of Fig. 2 we show simulation data for $E(\alpha, \beta)$ as obtained for $N = 10^6$ and $W = \tau = 0.00025T_0$. In the simulation we take $T_0 = 1$. In the experiment, for each event, the random numbers $A_{n,i} = 1, M$ select one out of four pairs $\{(\alpha_i, \beta_j) \mid i, j = 1, M\}$, where the angles α_i and β_i are fixed before the data is recorded. The data shown has been obtained by allowing for $M = 20$ different angles per station. Hence, forty random numbers from the interval $[0, 360[$ were used to fill the arrays $(\alpha_1, \ldots, \alpha_M)$ and $(\beta_1, \ldots, \beta_M)$. For each of the N events, two different random number generators were used to select the angles α_m and $\beta_{m'}$. The statistical correlation between m and m' was measured to be less than 10^{-6}. It is clear from the left panel of Fig. 2 that the simulation data for $E(\alpha, \beta)$ are in excellent agreement with quantum theory. Within the statistical noise, the simulation data (not shown) for the single-spin expectation values also reproduce the results of quantum theory.

Another way of demonstrating that the event-by-event simulation reproduces the results of quantum theory is to study the function $S(\alpha, \alpha', \beta, \beta')$. If

$$\max_{\alpha,\alpha',\beta,\beta'} S(\alpha, \alpha', \beta, \beta') > 2, \tag{16}$$

the two-particle correlations cannot be described by a product state or by the restricted class of locally causal models considered by Bell [16]. Violations of this inequality can be found by studying the function $S(\theta) \equiv S(\alpha, \alpha + 2\theta, \alpha + \theta, \alpha + 3\theta)$ [16]. Because of rotational invariance we may set $\alpha = 0$. In the right panel of Fig. 2, we present our simulation data for $S(\theta)$ obtained for $W = \tau = 0.00025T_0$ (red squares). For this value of W excellent agreement with quantum theory is observed. From Eq. (2), it is clear that by letting $W \to \infty$, we effectively discard the time-tag data in the calculation of the coincidences. The green crosses show the results for $S(\theta)$ obtained without using the time-tag data. This simulation data agrees with $E(\alpha, \beta) = -1 + 4\pi^{-1}|\alpha - \beta| \bmod \pi$, the correlation for a model considered by Bell [16] for which the inequality $|S(\alpha, \alpha', \beta, \beta')| \leqslant 2$ cannot be violated [16]. Thus, if the time-tag resolution τ and the time window W are sufficiently small, our model produces correlations that are undistinguishable from those of the singlet state. For $W \to \infty$ the correlations agree with those obtained from Bell's model.

As a check on the simulation results, we repeat the steps of the proof for fixed directions of observation given in our earlier work [15], not for the case of $S = 1/2$ particles but for particles carrying a polarization, and show that, in the limit $W \to 0$, the simulation model yields the two-particle correlation that is characteristic for a singlet state. Here, we give an alternative derivation that is more direct than the one given in Ref. [15].

Let us fix the orientations α and β and compute the ensemble average of the numerator of Eq. (3) for the ideal case of vanishing time-tag resolution τ. Note that the quantity that needs to be averaged is C_{xy}, not $E(\alpha, \beta)$. According to the specification of our model, taking the ensemble average means performing the integrals in

$$\sum_{x,y=\pm 1} xyC_{xy} = -\int_0^{2\pi} \frac{\mathrm{d}\xi}{2\pi} x_1 x_2 w\big(T(\alpha - \xi), T(\beta - \xi), W\big), \tag{17}$$

where $x_1 \equiv \operatorname{sign}(\cos 2(\xi - \alpha))$, $x_2 \equiv \operatorname{sign}(\cos 2(\xi - \beta))$, and

$$w(T_1, T_2, W) = \frac{1}{T_1 T_2} \int_0^{T_1}\int_0^{T_2} \Theta\big(W - |t - t'|\big)\,\mathrm{d}t\,\mathrm{d}t'. \tag{18}$$

The integral over ξ corresponds to the statement that the angles ξ_n are distributed uniformly over the interval $[0, 2\pi]$, the integral over t (t') corresponds to the statement that the time delay at station 1 (2) is distributed uniformly over the interval $[0, T(\xi_n - \alpha)]$ ($[0, T(\xi_n - \beta)]$). In the case of interest ($W \to 0$) we may replace $\Theta(W - |t - t'|)$ by $2W\delta(t - t')$. Then the integration over t and t' becomes trivial and gives

$$w(T_1, T_2, W) = \frac{2W}{\max(T_1, T_2)} + \mathcal{O}\big(W^2\big). \tag{19}$$

Repeating the same steps for the denominator of Eq. (3), yields (compare with Eq. (9) in Ref. [15])

$$E(\alpha, \beta) = -\frac{\int_0^{2\pi} x_1 x_2 \frac{\min(\sin^2 2(\xi-\alpha), \sin^2 2(\xi-\beta))}{\sin^2 2(\xi-\alpha)\sin^2 2(\xi-\beta)}\mathrm{d}\xi}{\int_0^{2\pi} \frac{\min(\sin^2 2(\xi-\alpha), \sin^2 2(\xi-\beta))}{\sin^2 2(\xi-\alpha)\sin^2 2(\xi-\beta)}\mathrm{d}\xi} = -\cos 2(\alpha - \beta), \tag{20}$$

demonstrating that our model *exactly* reproduces the correlation of a quantum system in the singlet state if $W \to \tau \to 0$, the regime that EPRB experiments aim to reach [23]. The averaging over the time-delays and the counting of coincidences introduces a weight

factor that gives rise to the singlet correlations: Although for each pair of particles, x_{n_i} and $t_{n,i}$ are Einstein-local quantities, the presence of this weight factor changes the functional form of the correlations such that the inequality $|S(\alpha, \alpha', \beta, \beta')| \leqslant 2$ may be violated.

6. Discussion

From the description of the algorithm, it is evident that $x_{n,i}$ and $t_{n,i}$ depend on the variable ξ_n that represents the polarization of the particles, and on $\gamma_{n,i}$, the angles of rotation by the electro-optic modulators. Furthermore, the event n cannot affect the data recorded for $n' \neq n$, implying that the algorithm simulates a causal process. In addition, $x_{n,i}$, $t_{n,i}$, or $\gamma_{n,i}$ do not depend on $\gamma_{n,j}$ for $i \neq j = 1, 2$. This implies that for each event, the numbers $x_{n,i}$ and $t_{n,i}$ do not depend on whatever action is taken at observation station j. Logically speaking, this means that the algorithm satisfies a more restrictive condition of locality than that of Einstein. For a logically local algorithm such as ours, the condition that the two systems be spatially separated is irrelevant.

It should be noted that Einstein's concept of local causality is different from the condition of local causality introduced by Bell [16]. The former applies to every individual fact (ontological level), the latter merely to the probability of a fact to occur (epistemological level) [28]. The model that we propose is a purely ontological model of the EPRB experiment. The expression for the coincidence (see Eq. (2)) cannot be written in terms of a product of two single-particle probabilities, an essential feature of the restricted class of models examined by Bell [16]. Hence there is no conflict.

Starting from the fact that an idealized EPRB experiment produces data sets $\{\Upsilon_1, \Upsilon_2\}$, we have constructed a computer simulation model that satisfies Einstein's conditions of local causality and reproduces the correlation $\widetilde{E}(\alpha, \beta) = -\cos 2(\alpha - \beta)$ that is characteristic for a quantum system in the most entangled state, the singlet state. Salient features of our model are that it generates the data set $\{\Upsilon_1, \Upsilon_2\}$ event-by-event, uses integer arithmetic and elementary mathematics to analyze the data, does not rely on concepts of probability theory and quantum theory, and provides a simple, rational and realistic picture of the mechanism that yields correlations that agree with those of a quantum system in the singlet state.

The key ingredient of our algorithm is the use of the same expression for the coincidences as the one employed in the experiment, see Eq. (2), which depends on the settings of the variable polarizers, on the difference of the time tags of the detection signals in both observation stations and on the time window W. The idea that the coincidence counts do not only depend on the experimental settings but also on the time-dependencies that are intrinsic to the measuring equipment has been considered earlier [14,20,21,29]. As W is solely used in the data analysis procedure, carried out (long) after the experiment has finished, no definite statement about the source (or total experimental set-up) can be made from the obtained correlations.

According to our model, we expect that numerical agreement between the experimental results and quantum theory improves if the time-tag resolution τ and the time window W are made smaller and smaller. The time delays induce uncertainty about which photons belong to a photon pair. Disregarding the time-tag data yields results that disagree with quantum theory and with experiment but agree with the models considered by Bell. Our results show that for sufficiently small time-tag resolution, increasing the time window changes the nature of the two-particle correlations. This prediction can easily be tested by using available experimental data. Finally, we would like to mention that although the simulation model that we have proposed demonstrates that it is possible to perform an event-by-event simulation of EPRB experiments, it is not is unique in this respect.

Appendix A. Demo computer program

```
!
! FORTRAN 90 program to simulate an Einstein-Podolsky-Rosen-Bohm experiment
!
        integer,parameter :: M=10 ! number of angles at station 1 and 2 + 1
        real :: angles1(0:M),angles2(0:M) ! angles for station 1 and 2
        integer :: Notiming(0:1,0:1,0:M,0:M) ! counts without using time-tag data
        integer :: Ntiming(0:1,0:1,0:M,0:M)  ! counts using time-tag data
        integer :: iseed (1) ! seed for the random number generator

! model parameters
        N=100000    ! Number of events generated by the source for each pair of angles
        tau=0.0025  ! time-tag resolution in units of T0 = 1, see text
        k=1         ! time window W = k * tau
        d=2         ! parameter for the time-tag model
! initialization of the random number generator
        iseed=9896432
        call random_seed(put=iseed)
! initialize tables of angles of observation
        pi=-1
        pi=acos(pi)
```

```
        twopi=2*pi
        pi2=pi/2
        do i=0,M         ! use the random number generator to pick
        call random_number(r0)  ! angles from [0,pi]
        angles1(i)=r0*pi
        enddo
        do i=0,M         ! use the random number generator to pick
        call random_number(r0)  ! angles from [0,pi]
        angles2(i)=r0*pi
        enddo
! clear counters
        Notiming=0
        Ntiming=0

! generate events
        do i=1,N*M*M
! pick a polarization
        call random_number(r0)
        x1=twopi*r0 ! polarization of particle 1
        x2=x1+pi2   ! polarization of particle 2
! station 1
        call random_number(r0)
        i1=(M+1)*r0 ! pick an angle at station 1
        c1=cos(2*(x1-angles1(i1)))
        if(c1>0) then ! sign(cos(...))
        j1=0 ! <=> -1 event
        else
        j1=1 ! <=> +1 event
        endif
        call random_number(r0)
        k1=ceiling((1-c1*c1)**(d/2)*r0/tau) ! delay time
! station 2
        call random_number(r0)
        i2=(M+1)*r0 ! pick an angle at station 2
        c2=cos(2*(x2-angles2(i2))) !
        if(c2>0) then ! sign(cos(...))
        j2=0 ! <=> -1 event
        else
        j2=1 ! <=> +1 event
        endif
        call random_number(r0)
        k2=ceiling(abs(1-c2*c2)**(d/2)*r0/tau)  ! delay time
! count
        Notiming(j1,j2,i1,i2)=Notiming(j1,j2,i1,i2)+1 ! Model without timing (W > T0)
        if(abs(k1-k2)<k) then ! model with timing
        Ntiming(j1,j2,i1,i2)=Ntiming(j1,j2,i1,i2)+1
        endif
        enddo ! do i=1,N*M*M ! generate events
!
! Print results to standard output
! Redirect the output of this program to the file (e.g. a.out > x )
! and use a scatter (x,y) plot to display the data.
! GNUPLOT users: gnuplot> plot "x" u 1:3 w p, "x" u 1:2 w p, "x" u 1:4 w p
!
        write(6,*) &
"# a-b,-cos(2*(a-b)),E(a,b,W->0),E(a,b,W>T0),E1(a,W->0),E2(b,W->0),f(a,b),E1(a,W>T0),E2(b,W>T0):"
        write(6,*) &
"#-------------------------------------------------------------------------------------------"
        do i2=0,M
        do i1=0,M
        angle=angles1(i1)-angles2(i2)
! expectations without using time-tag information (Bell-type model)
```

```
! denominator of (2)
        r0=Notiming(0,0,i1,i2)+Notiming(1,1,i1,i2)+Notiming(0,1,i1,i2)+Notiming(1,0,i1,i2)
! numerator of (2)
        r1=Notiming(0,0,i1,i2)+Notiming(1,1,i1,i2)-Notiming(0,1,i1,i2)-Notiming(1,0,i1,i2)
        r2=Notiming(0,0,i1,i2)+Notiming(0,1,i1,i2) ! single particle counts
        r3=Notiming(0,0,i1,i2)+Notiming(1,0,i1,i2) ! single particle counts
        if(r0>0) then
        r1=r1/r0
        r2=r2/r0
        r3=r3/r0
        endif
! expectations using time-tag information
! denominator of (2)
        t0=Ntiming(0,0,i1,i2)+Ntiming(1,1,i1,i2)+Ntiming(0,1,i1,i2)+Ntiming(1,0,i1,i2)
! numerator of (2)
        t1=Ntiming(0,0,i1,i2)+Ntiming(1,1,i1,i2)-Ntiming(0,1,i1,i2)-Ntiming(1,0,i1,i2)
        t2=Ntiming(0,0,i1,i2)+Ntiming(0,1,i1,i2) ! single particle counts
        t3=Ntiming(0,0,i1,i2)+Ntiming(1,0,i1,i2) ! single particle counts
        if(t0>0) then
        t1=t1/t0
        t2=t2/t0
        t3=t3/t0
        endif
        write(6,'(1x,9e11.3)') 180*angle/pi,-cos(2*angle),t1,r1,t2,t3,t0/N,r2,r3
        enddo
        enddo
        end
```

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