Computer Simulation of Einstein-Podolsky-Rosen-Bohm Experiments with Photons

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http://www.compphys.net/dlm

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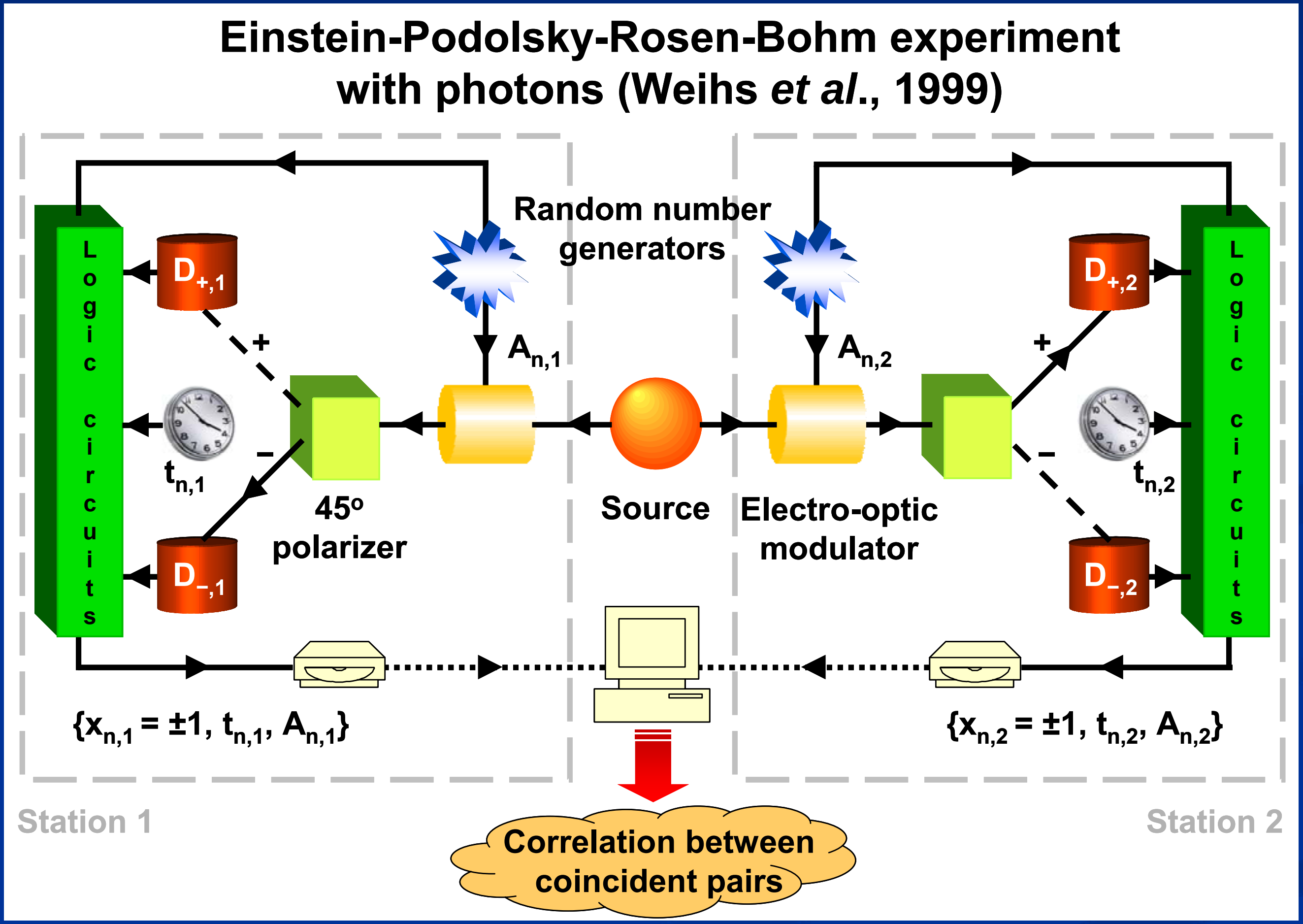
Introduction

- Computer simulation is complementary to theory and experiment*

- Conventional approach:
  - Start from the equations of physics
  - Use or invent an algorithm that solves these equations
  - Interpret the results, compare with theory or experiment
  - If necessary, refine the model and go to step 2
- What if there are no "equations of physics"?

*D.P. Landau and K. Binder, *A guide to Monte Carlo Simulation in Statistical Physics*, Cambridge Univ. Press (2000)

Example: Real Einstein-Podolsky-Rosen-Bohm experiments



* G. Weihs, T. Jennewein, C. Simon, H. Weinfurther, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998)

Quantum theory for the EPRB experiment

- Single system of two *S*=1/2 particles
- The whole experiment is described by a singlet (total spin zero) state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2\right)$$

- A simple calculation shows that

$$E_1(\mathbf{a},\mathbf{b}) = E_1(\mathbf{a}) = \langle\Psi|\boldsymbol{\sigma}_1\cdot\mathbf{a}|\Psi\rangle = 0$$

$$E_2(\mathbf{a},\mathbf{b}) = E_2(\mathbf{b}) = \langle\Psi|\boldsymbol{\sigma}_2\cdot\mathbf{b}|\Psi\rangle = 0$$

$$E(\mathbf{a},\mathbf{b}) = \langle\Psi|\boldsymbol{\sigma}_1\cdot\mathbf{a}\;\boldsymbol{\sigma}_2\cdot\mathbf{b}|\Psi\rangle = -\mathbf{a}\cdot\mathbf{b}$$

If QT is used to ``explain" data → EPR paradox

Fundamental limitation of quantum theory

- We can use quantum theory to compute probability distributions (interference patterns) but quantum theory cannot model the process in terms of the individual events that we observe in a real experiment
  - Not a contradiction: Quantum theory does not describe individual events, only the collective result of many events

- Reconciling the formalism of quantum theory with the experimental fact that each observation yields a definite outcome is called the quantum measurement paradox and is the central, most fundamental problem in the foundations of quantum theory
    - D. Home, *Conceptual Foundations of Quantum Physics*, Plenum Press, New York (1997)

Fundamental question

- Can we model the event-by-event processes observed in real experiments and reproduce the same statistical answers of experiments and quantum theory (without first solving the Schrödinger equation) ?
- After 100 years of hard work: All attempts to extend quantum theory have failed
  - Quantum measurement paradox
  - Prevailing logic in physics: Don’t ask this question

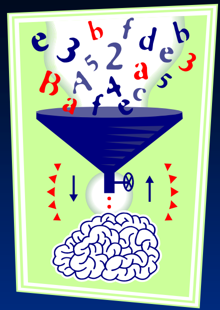
This talk is not about interpretations of quantum theory

What if we ask "the question"?

- Strategy: Stick to the data (= single events) that is provided by experiment and look for processes that generate these events such that the collective outcome agrees with quantum theory

$$\Upsilon_{N,i} = \left\{ x_{n,i} = \pm 1, t_{n,i}, A_{n,i} \mid n = 1, \ldots, N \right\} \quad , \quad i = 1,2$$

  - Quantum theory has nothing to say about individual events anyway
    - Niels Bohr: "There is no quantum world. There is only an abstract quantum mechanical description. It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature."



Data analysis (1)

- In any practical realization of an (EPR-Bohm) experiment, it is necessary to have a criterion that decides which particles form a pair and which particles do not
- In EPR-Bohm experiments, coincidence in time $|t_{n,1} - t_{n,2}| < W$ is used to define a pair*
  - W is a time window, chosen by the experimenter

[#] C.A. Kocher and E.D. Commins, Phys. Rev. Lett. 18, 575 (1969)

[*] G. Weihs, T. Jennewein, C. Simon, H. Weinfurther, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998)

Data analysis (2)

- After all data has been collected, compute the two-particle coincidences*

$$C_{xy}(\alpha,\beta)=\sum_{n=1}^{N}\delta_{x,x_{n,1}}\delta_{y,x_{n,2}}\delta_{\alpha,A_{n,1}}\delta_{\beta,A_{n,2}}\Theta(W-|t_{n,1}(x,\alpha)-t_{n,2}(y,\beta)|)$$

  - x,y = ++,--,+-,-+ (+ ⇔ +1, - ⇔ -1)
  - α,β : rotation angles ⇔ setting of the electro-optic modulators 1 and 2
- Compute the two-particle correlation*

$$E(\alpha,\beta)=\frac{C_{++}(\alpha,\beta)+C_{--}(\alpha,\beta)-C_{+-}(\alpha,\beta)-C_{-+}(\alpha,\beta)}{C_{++}(\alpha,\beta)+C_{--}(\alpha,\beta)+C_{+-}(\alpha,\beta)+C_{-+}(\alpha,\beta)}$$

* G. Weihs, T. Jennewein, C. Simon, H. Weinfurther, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998)

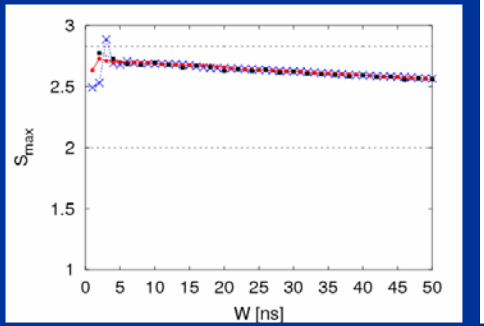
Real EPRB experiment

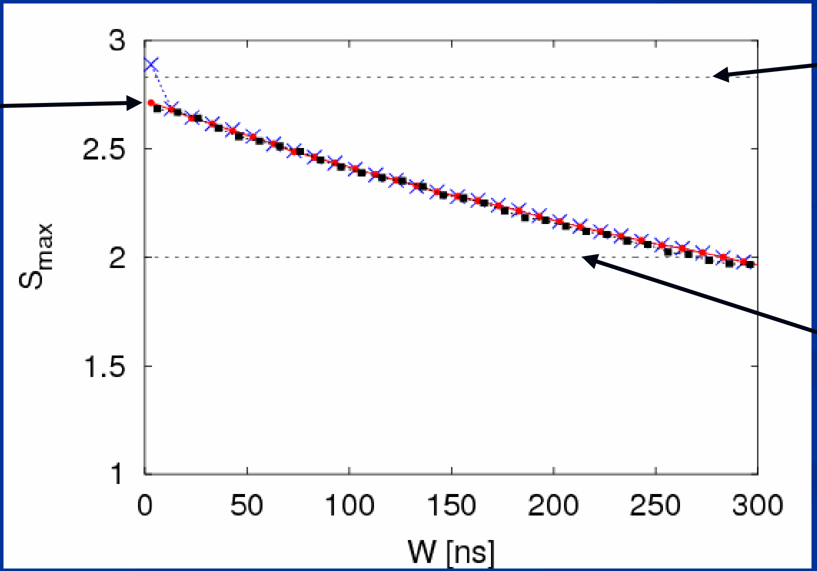
- Our analysis of experimental data of Weihs et al. using three different methods
  - http://www.quantum.at/research/photonentangle/bellexp/data.html

$$S_{max} \equiv E(a,c) - E(a,d) + E(b,c) + E(b,d)$$

Experiment: $a = 0, b = \pi/8, c = \pi/4, d = 3\pi/8$

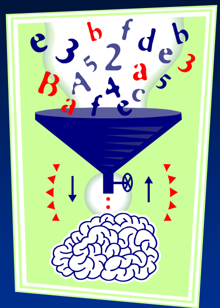
"Best" value cited in literature: 2.73 (Weihs et al.)





Upper bound for a system of two S=1/2 particles

Upper bound for a system of two uncorrelated S=1/2 particles



A Solution (1)

Listen to what the data has to say,
not what people say about the data

- Start from the observation that experiment generates data sets[#]

$$\Upsilon_{N,i} = \left\{ x_{n,i} = \pm 1, t_{n,i}, A_{n,i} \,|\, n = 1, \ldots, N \right\} \quad , \quad i = 1, 2$$

  - Main rule of the game: Einstein's criterion of local causality* (≠ Bell's notion of locality)
    - "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former"

[#] G. Weihs, T. Jennewein, C. Simon, H. Weinfurther, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998)

* P.A. Schilpp, Ed., "Albert Einstein, Philospher-Scientist, Tudor, NY (1949)

A Solution (2)

- Simulation model:
  - Particle *i* =1,2 carries a vector $\mathbf{S}_{n,i} = (-1)^{i+1}(\cos\xi_n, \sin\xi_n)$
  - The electro-optic modulator *i* rotates this vector by α_i
  - The polarizer *i* directs the particle to the detector $x_{n,i} = \pm 1$

    $$p(x_{n,i} \mid \xi_n - \alpha_i) = \cos^2(\xi_n - \alpha_i)$$ ⟹ Malus law

  - The modulator+polarizer causes a time delay

    $$0 \le t_{n,i} - t_0 \le T \,|\sin 2(\xi_n - \alpha_i)|^d$$

  - Correlations are calculated in exactly the same manner as in experiment

Satisfies Einstein’s criteria of local causality and realism

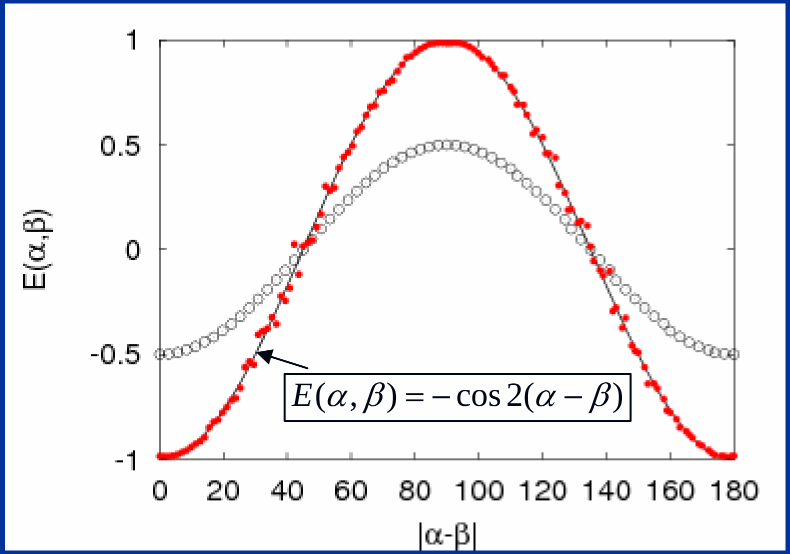
Simulation model: Free parameters

- Distribution of vectors $\mathbf{S}_{n,i} = (-1)^{i+1}(\cos\xi_{n,i}, \sin\xi_{n,i})$
- Time-delay exponent *d*
- Coincidence window W / τ

- Maximum time delay T/τ
- Number of events *N*

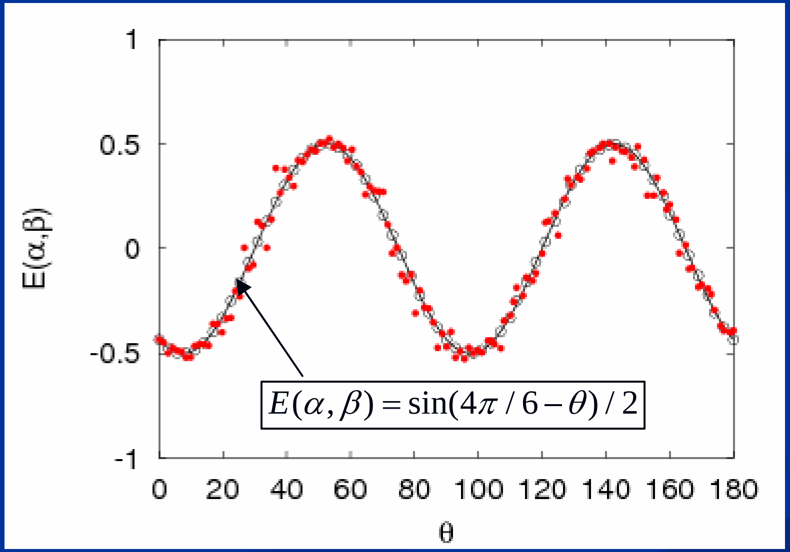
Simulation results

- $\xi_{n,2}=\xi_{n,1}+\pi/2$
  - uniform distribution
  - $N = 10^6$, $W = \tau = 0.00025T$



○ $d=0$ → Text book "Bell" model
● $d=4$ → Quantum theory: singlet state

- $\xi_{n,2}=\xi_{n,1}+\pi/2$
  - $\xi_{n,1}=\pi/6$, $\alpha=\theta$, $\beta=\theta+\pi/4$
  - $N = 10^6$, $W = \tau = 0.00025T$



○ $d=0$ → Quantum theory: product state
● $d=4$ → Quantum theory: product state

Summary of results

- Our event-by-event simulation models for the EPR-Bohm experiments reproduce all the results of quantum theory for a system of two S=1/2 particles*
  - Models strictly satisfy Einstein's conditions of local causality
  - Rigorous proofs for integer *d*
  - For $d = 0$ or $W \to \infty$ (⇔ removing the time-tag data), we recover the results of a model considered by Bell
  - Textbook "EPR paradox" is the result of analyzing experiments in terms of (Bell-type) models that do not account for all essential experimental data

*De Raedt, Keimpema, De Raedt, Michielsen, Miyashita, Eur. Phys. J. B 53, 139 (2006);
(De Raedt)^2, Michielsen, Comp. Phys. Comm. 176, 642 (2007);
(De Raedt)^2 , Michielsen, Keimpema, Miyashita, J. Comp. Theor. Nanosci. (in press)
(De Raedt)^2 , Michielsen, Keimpema, Miyashita, J. Phys. Soc. Jpn. (in press)

Conclusion

- We have invented a systematic, modular procedure to construct locally causal, classical (non-Hamiltonian) dynamical systems that can be used for a deterministic or pseudo-random (unpredictable) event-by-event simulation of real-time quantum phenomena
  - Evidence that our event-by-event simulation approach works (http://www.compphys.net/dlm):
    - Single-photon Mach-Zehnder interferometers
    - Universal quantum computation
    - Quantum cryptography
    - Wheeler's delayed choice experiment
    - Quantum eraser, single-photon quantum optics in general
    - EPRB experiments with non-orthogonal detection planes
- Our simulation approach allows the modeling of nanoscale processes on the level of individual events without using concepts of quantum theory

$P(A|BabC)=P(A|aC)$

Local causality according to J.S. Bell

- According to Bell, in a locally causal theory, if *b* has no causal effect on *A* then $P(A|bZ)=P(A|Z)$
  - J.S. Bell, “Speakable and unspeakable in quantum mechanics”, p.54
- Example (E.T. Jaynes, 1989): Consider a vase with one red and 1 white ball. A blind monkey draws the balls.
  - *A*: First draw yields a red ball, *b*: Second draw yields a red ball
  - Experiment 1: Show the result of the first draw $\longrightarrow$ $P(b|AZ)=0$
  - Experiment 2: Do not show the result of the first draw
    - As the second draw cannot have a causal effect on the first draw, according to Bell, in a locally causal theory, we must have
      - Experiment 2: $P(A|bZ)=P(A|Z)=1/2$?
    - Correct application of probability theory (= common sense)

      $$P(Ab|Z)=P(A|bZ)P(b|Z)=P(b|AZ)P(A|Z) \longrightarrow P(b|AZ)=P(A|bZ)$$
      - Experiment 2: $P(A|bZ)=0$

$P(A \mid BabC) = P(A \mid aC)$

Local causality according to J.S. Bell

- Bell did not seem to have realized that the absence of causal influence does not imply logical independence

Logical independence

$\neq$

Physical independence

  - First logic then physics
- Bell's extension of Einstein's event-based notion of locality to probabilistic theories leads to logical inconsistencies
    - A vase with a red and a white ball is "nonlocal"?
- Bell's "theorem" is irrelevant for (quantum) physics